Analyticity Requirement for Regge Poles and Backward Unequal-Mass
Scattering II

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ABSTRACT

We evaluate exactly the modified Regge amplitude for the backward unequal-mass scattering. This

gives the Regge behavior \( u'' \) at \( s = 0 \) and in its neighborhood. We then consider a possibility that a fixed

pole contribution is completely eliminated by a counteracting Regge amplitude. It is shown that, if

such an amplitude is added, the total Regge amplitude vanishes at \( s = 0 \).

In a previous paper, \(^{12}\) a Regge amplitude satisfying the Mandelstam representation is

constructed for the backward unequal-mass scattering. Based on the two lowest-order coefficients in a

power series expansion in \( s \), it was shown that the leading term gives the Regge behavior \( u'' \) at \( s = 0 \) and in its neighborhood including the region in which the cosine of the backward scattering angle is bounded. It was shown also that the coefficients of the non-leading terms do not give rise to a singularity of the Regge amplitude.

In this paper, we obtain the exact \( s \) dependence of the asymptotic form of the Regge amplitude

for large \( u \), that is, we compute the coefficients of the \( u'' \), \( u''' \), and \( u^{(0)'} \) terms exactly.

We confirm the assertions made in the previous paper.

\(^{13}\) We then consider a possibility of eliminating the fixed power term \( u^{(0)} \) by introducing another Regge pole with \( u^{(0)'} \), \( u^{(0)'} \), and \( u^{(0)'} \) terms. It is shown that this elimination procedure leads to a vanishing total Regge amplitude at \( s = 0 \).

In the previous paper, \(^{13}\) it was shown that the analyticity approach leads to the following expression for the modified Regge amplitude.

\[
K(s, u) = \frac{1}{\pi} \int_{s_0} Im R(s', u) ds',
\]

(1)

where

\[
R(s', u) = -\pi\left( -q'' \right)^2 \frac{P_{\pi N}}{2q''^2} \times \left( -1 - \frac{u - r''/s'}{r''} \right),
\]

\[
q'' = \frac{[s' - (m + \mu)^2](s' - (m + \mu)^2)}{4s'},
\]

and

\[
s_0 = (m + \mu)^2.
\]

m and \( \mu \) are the nucleon and pion masses respectively. As in all previous papers on this subject we are dealing here with pion-nucleon backward scattering.

The Regge amplitude \( R(s', u) \), in addition to the physical cut, has a cut running from \( s' = 0 \) to \( r''/u \) in the complex \( s' \) plane. Therefore the above dispersion integral can be replaced by a contour integral which encloses the pole of the integrand at \( s' = s \) and a cut running from 0 to \( r''/u \) counterclockwise.

\[
K(s, u) = \frac{1}{\pi} \int_C ds' \frac{R(s', u)}{s' - s}.
\]

(2)

\( C \) is the contour described above. We can choose this contour in such a way that the following conditions are satisfied:

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(1) $a(s')$ and $\gamma(s')$ are analytic within the contour.

(2) $q^s = r^s/4s'$ along the contour.

(3) $P_{s'u'} \left( 1 - \frac{2us'}{r^2} \right) = \frac{1}{\sqrt{\pi}} \frac{\Gamma[a(s') + 1/2]}{\Gamma[a(s') + 1]} \times \left[ u^{s'} - \frac{a(s')r^2}{2s'} u^{s'-1} \right]$ along the contour.

Then

$$R(s', u') = -\sqrt{\pi} \gamma(s') \frac{\Gamma[a(s') + 1/2]}{\Gamma[a(s') + 1]} \left[ u^{s'} - \frac{a(s')r^2}{2s'} u^{s'-1} \right].$$

(4)

According to the above expression for $R(s', u')$, the integrand of Eq. (2) has poles at $s'=s$ and $s'=0$. Now, by taking residues we obtain

$$K(s, u) = \frac{\sqrt{\pi} \gamma(s)}{\Gamma[a(s') + 1]} \left[ u^{s'} - \frac{a(s')r^2}{2s'} u^{s'-1} \right].$$

The leading term in Eq. (5) is $u^{s'}$ as has been expected. The above $K(s, u)$ also gives a fixed-pole contribution $\frac{1}{s} u^{s'-1}$. This term has a pole at $s=0$. But this singularity is canceled by the $\frac{1}{s} u^{s'-1}$ term which also exists in the right hand side of Eq. (5). Unfortunately, Freedman et al. did not realize that these poles cancel each other at $s=0$ and asserted the existence of a daughter trajectory which will remove the singularity of the fixed power term. We may mention further that $K(s, u)$ of Eq. (5) reduces to that of the previous paper in the small $s$ limit.

It is by now very clear that the daughter trajectory of Freedman et al. type does not exist. But, let us consider a possibility of eliminating completely the fixed-pole contribution by introducing a counteracting Regge pole, that is, we construct a new amplitude $\tilde{K}(s, u)$ with the parameters $\tilde{a}(s)$ and $\tilde{\gamma}(s)$, add to the original amplitude:

$$K_{tot}(s, u) = K(s, u) + \tilde{K}(s, u),$$

and insist that $K_{tot}(s, u)$ contains no fixed pole terms. Then from Eq. (5) it is clear that $\tilde{a}(0) = \tilde{a}(0)$. In other words, the two trajectories must cross each other at $s=0$. This further leads to $\tilde{\gamma}(0) = -\tilde{\gamma}(0)$. Then $K_{tot}(s, u) = 0$ at $s=0$. This total amplitude does not have a Regge behavior at $s=0$.

In this paper, we first evaluated exactly the coefficients of the $u^{s'}$, $u^{s'-1}$ and $u^{s'-1}$ terms. Using these coefficients we have shown that an attempt to eliminate the fixed power term by another Regge pole leads to a vanishing total amplitude at $s=0$.

**REFERENCES**

(1) Y. S. Kim and M. Resnikoff, Phys. Rev. 169, Sec. 5 (May, 1968) (to be published)