

The symmetry between $\cos \chi$ and $\sin \chi$ is well known. Let us now consider another Poincaré sphere where $\cos \chi$ is replaced by $\sin \chi$. Then the density matrix and entropy become

$$\rho'(\chi) = \frac{1}{2} \begin{pmatrix} 1+f' & 0 \\ 0 & 1-f' \end{pmatrix},$$

$$S' = - \left(\frac{1+f'}{2} \right) \ln \left(\frac{1+f'}{2} \right) - \left(\frac{1-f'}{2} \right) \ln \left(\frac{1-f'}{2} \right),$$

respectively, with

$$f' = \frac{\sqrt{(a^2 - b^2)^2 + 4(ab)^2 \sin^2 \chi}}{a^2 + b^2},$$

which takes its minimum and maximum values when $\chi = 0$ and $\chi = 90^\circ$ respectively.

Indeed, S and S' move in opposite directions as χ changes. Thus we are led to consider their addition ($S + S'$). The question is whether it remains constant. The answer is No. On the other hand, the determinant of the first coherency matrix is $(ab)^2 \sin^2 \chi$, and it is $(ab)^2 \cos^2 \chi$ for the second determinant. Thus the addition of these two determinants

$$(ab)^2 \sin^2 \chi + (ab)^2 \cos^2 \chi = (ab)^2.$$

is independent of the decoherence angle χ .

While the determinant of is a Lorentz-invariant quantity, there could be a larger group which will change the value of the determinant, thus the decoherence angle. This question has been addressed in the literature.

Then there comes the issue of Feynman's rest of the universe. In his book on statistical mechanics, Feynman divides the quantum universe into two systems, namely the world in which we do physics, and the rest of the universe beyond our scope.

However, we could gain a better understanding of physics if we can construct a model of the rest of the universe which can be explained in terms of the physical laws applicable to the world which we observe.