

We now have the four-vector $(\mathbf{S}_0, \mathbf{S}_3, \mathbf{S}_1, \mathbf{S}_2)$, which is Lorentz-transformed like the space-time four-vector (t, z, x, y) or the energy-momentum four vector. This Stokes four-vector has a three-component subspace $(\mathbf{S}_3, \mathbf{S}_1, \mathbf{S}_2)$, which is like the three-dimensional Euclidean subspace in the four-dimensional Minkowski space. In this three-dimensional subspace, we can introduce the spherical coordinate system with

$$R = \sqrt{S_3^2 + S_1^2 + S_2^2}$$

and

$$R = \frac{1}{2} \sqrt{(a^2 - b^2)^2 + 4(ab)^2 \cos^2 \chi}.$$

with

$$S_3 = \frac{a^2 - b^2}{2}.$$

and

$$S_1^2 + S_2^2 = (ab)^2 (\cos \chi)^2.$$

This spherical picture of is traditionally known as the Poincaré sphere. Its radius R takes its maximum value S_0 when $\chi = 0^\circ$. It decreases and reaches its minimum value, S_3 , when $\chi = 90^\circ$.

Under the Lorentz transformation, all four Stokes parameters are subject to change. The maximum radius S_0 and the minimum radius S_3 do not remain invariant. The radius R is also subject to change. While Lorentz transformations shake up these parameters, is there any quantity remaining invariant?

Let us go back to the four-momentum matrix

$$P = \begin{pmatrix} E + p_z & p_x - ip_y \\ p_x + ip_y & E - p_z \end{pmatrix},$$

Its determinant is m^2 and remains invariant. Likewise, the determinant of the coherency matrix of Eq.?? should also remain invariant. The determinant in this case is

$$S_0^2 - R^2 = (ab)^2 \sin^2 \chi.$$

This quantity remains invariant.

While the decoherence parameter is not fundamental is influenced by environment, it plays the same mathematical role as in the particle mass which remains as the most fundamental quantity since Isaac Newton, and even after Einstein.