Space-time Entanglement

We are familiar with the Lorentz boost

$$z \to z\ \cosh\eta + t \sinh\eta, \qquad t \to z\ \sinh\eta + t\cosh\eta.$$

This is not a rotation, but this transformation can be separated if we use Dirac's light-cone coordinate system:

$$(z+t) \to e^{\eta}(z+t), \qquad (z-t) \to e^{-\eta}(z-t).$$

If we apply this transformation to Dirac's oscillator wave function

$$\exp\left\{-\frac{1}{2}\left(z^2+t^2\right)\right\} = \exp\left\{-\frac{1}{4}\left[(z+t)^2+(z-t)^2\right]\right\},$$

the transformed wave function becomes

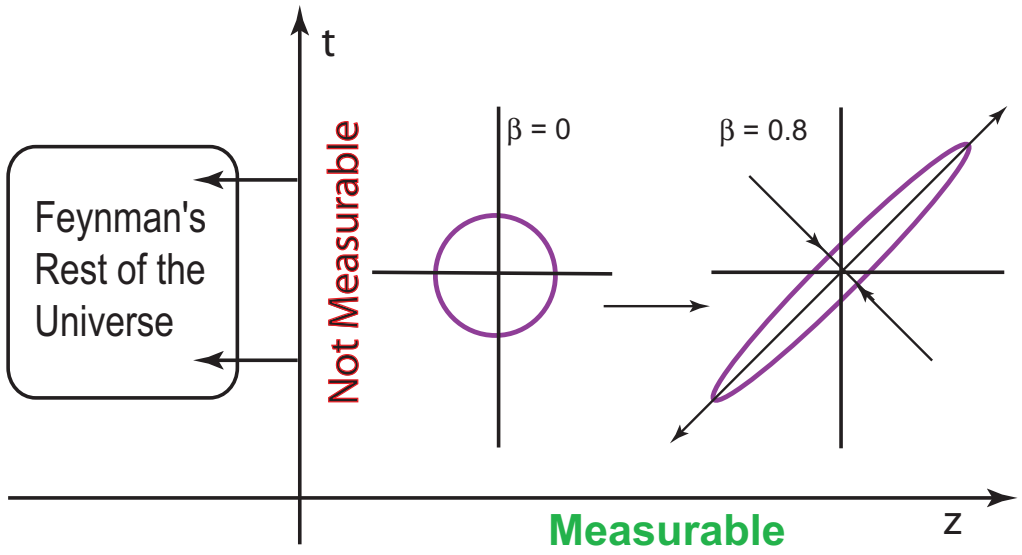
$$\exp\left\{-\frac{1}{4}\left[e^{-2\eta}(z+t)^2+e^{2\eta}(z-t)^2\right]\right\}.$$

This transformed function is no longer separable in z and t variables. In terms of the the complete set of the oscillator wave functions $\phi_n(z)$ and $\phi_n(t)$, the above expression can be written as

$$\frac{1}{\cosh\eta}\sum_n (\tanh\eta)^n \phi_n(z)\phi_n(t).$$

This is an entangled state in the space and time variables. The t variable is tells the time separation (a the Bohr radius tells the space separation), and is not the propagation time.

The Lorentz boost entangles the space and time variables. We can see what we did above in the following figure.



What is Feynman's rest of the universe? Another interesting subject!