

Feynman and Dirac bonded by Squeezed States

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University of Maryland

U.S.A.

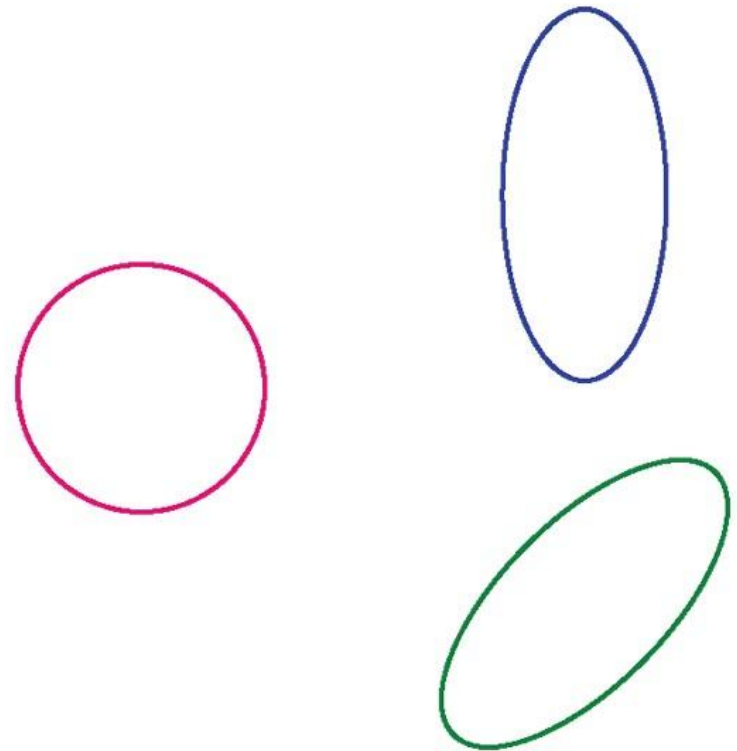
Dirac and Feynman in Poland (1962)

- Dirac: construct a beautiful mathematics.
- Feynman: mathematical instrument that will produce numbers which can be compared with what we observe in the real world.



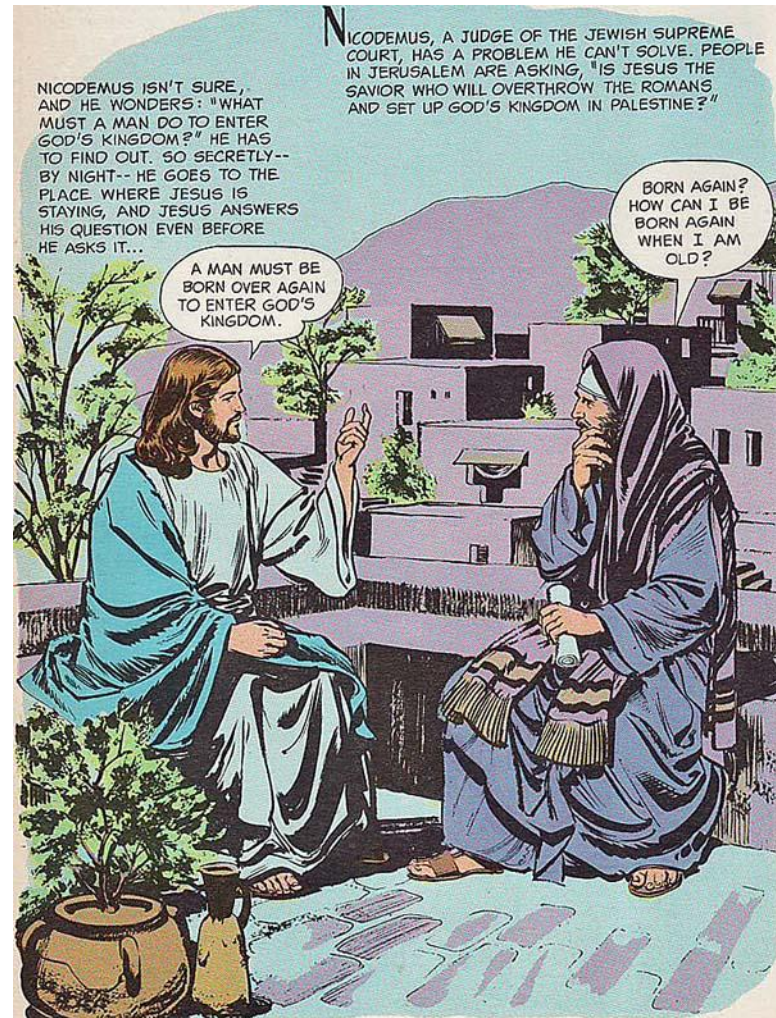
How?

- 2-by-2 matrices.
rotations and
squeeze
- 2-d Gaussian
distribution



Audience with Dirac in 1962

Like
Nicodemus
asking
Jesus

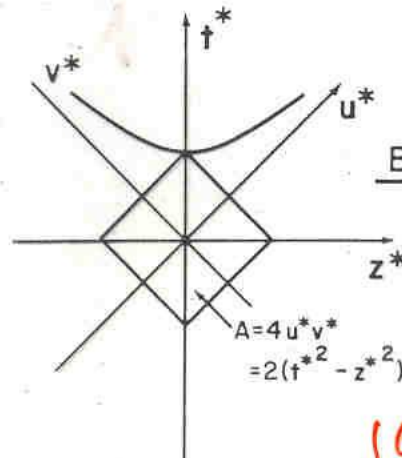
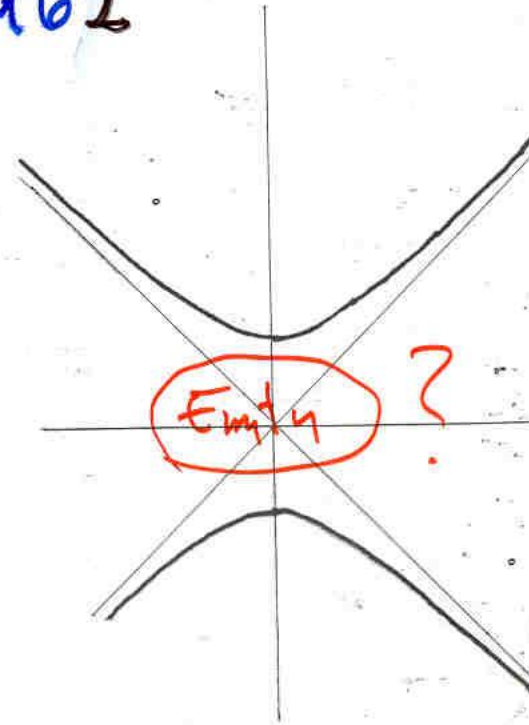


I asked Dirac
what I should
do in Physics.

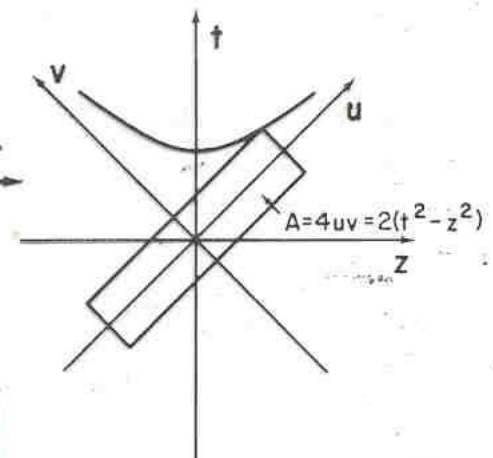
Study Lorentz
covariance.
Americans
should study
more about
this subject.



1962



BOOST



1949

Five Books of Dirac

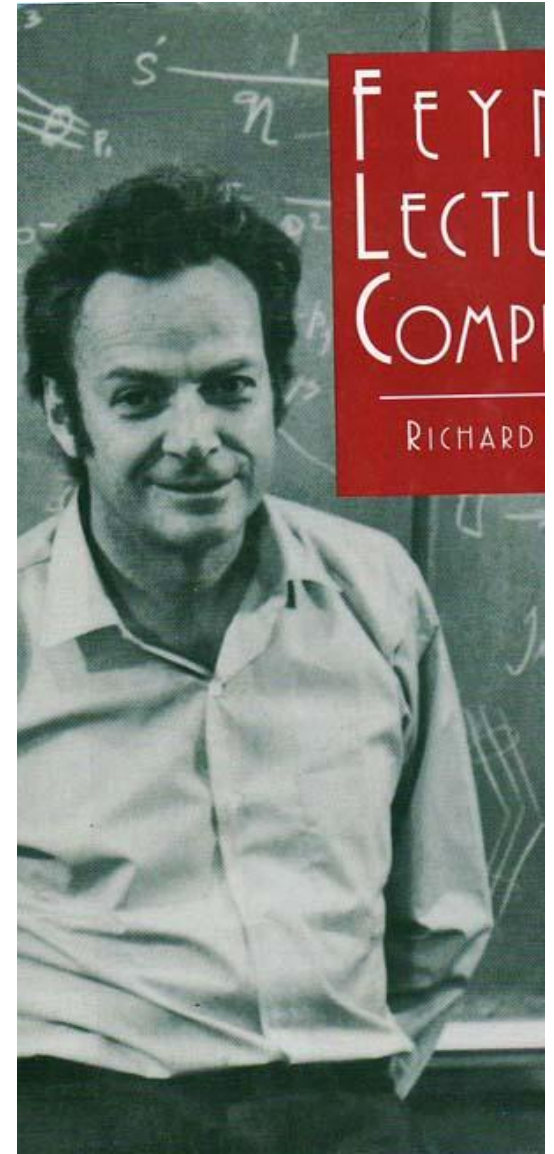
- First Book. 1927 paper on time-energy uncertainty – c-number
- Second Book. 1945 paper on harmonic oscillators. Gaussian time distribution.
- Third Book. 1949 paper on light-cone coordinate system
- Fourth Book: 1963 paper on two-oscill. system— **two mode squeezed states.**
- Fifth Book. To be written.

Why do we need the Fifth Book?

- Dirac's papers are like poems, but contain no figures. Translate those poems into cartoons.
- Those papers lack continuity. Dirac does not make reference to his earlier papers.
- **Dirac does not make difference between time and time interval.**

Feynman's Interpretation of Physics

The adventure of our science of physics is a perpetual attempt to recognize that the different aspects of nature are really different aspects of the same thing.



- Feynman wrote 150 papers.
- He wanted to pack them into one paper.
- IWe can combine three of them into one paper..

1969 paper on partons.

1971 paper on covariant oscillators

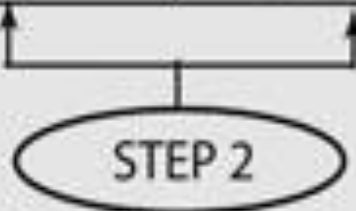
1972 book. Rest of the universe.

Concept of entanglement.

**It then becomes the Fifth Book of
Paul A, M Dirac**

1970 Feynman's Crazy Talk

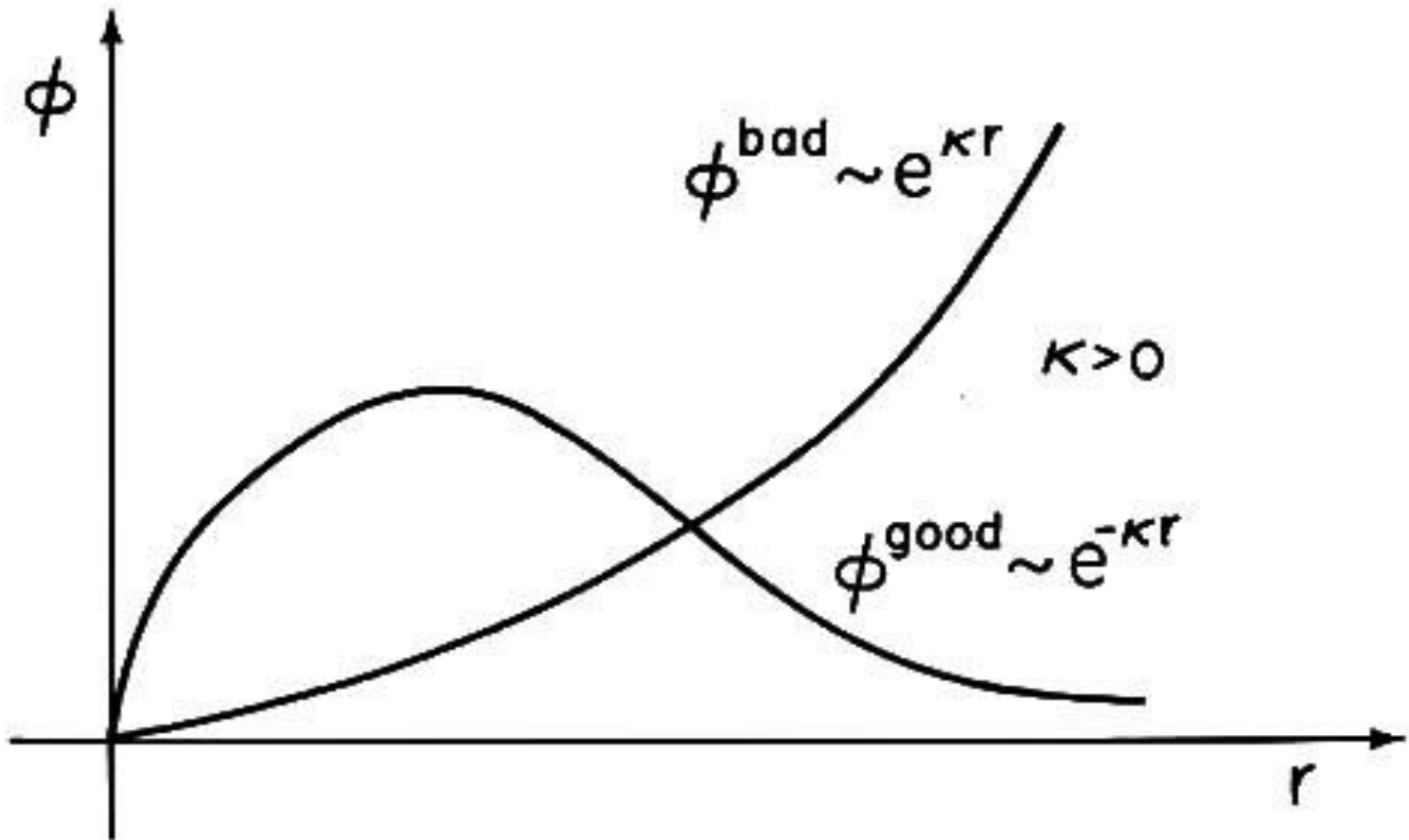


Scattering	Bound States	Space/Time
COMETS	PLANETS	GALILEI
NEWTON		
	BOHR	
HEISENBERG, SCHRÖDINGER		
FEYNMAN		EINSTEIN
		

Feynman said!

- **For Scattering problems, use Feynman diagrams.**
- **For bound-state problems, use harmonic oscillators.**
- **However, there were no well-known Lorentz-covariant oscillators in 1970.**
- **People said Feynman was absolutely crazy.**
- **But, I became excited about what he said, and studied his physics systematically since then. I also became crazy.**

1965. Feynman diagrams do not guarantee localization.



Based on Feynman's 1970 Talk

PHYSICAL REVIEW D

VOLUME 3, NUMBER 11

1 JUNE 1971

Current Matrix Elements from a Relativistic Quark Model*

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Lauritsen Laboratory of Physics, California Institute of Technology, Pasadena, California 91109

(Received 17 December 1970)

A relativistic equation to represent the symmetric quark model of hadrons with harmonic interaction is used to define and calculate matrix elements of vector and axial-vector currents. Elements between states with large mass differences are too big compared to experiment, so a factor whose functional form involves one arbitrary constant is introduced to compensate this. The vector elements are compared with experiments on photoelectric meson production, K_{12} decay, and $\omega \rightarrow \pi\gamma$. Pseudoscalar-meson decay widths of hadrons are calculated supposing the amplitude is proportional (with one new scale constant) to the divergence of the axial-vector current matrix elements. Starting only from these two constants, the slope of the Regge trajectories, and the masses of the particles, 75 matrix elements are calculated, of which more than $\frac{2}{3}$ agree with the experimental values within 40%. The problems of extending this calculational scheme to a viable physical theory are discussed.

INTRODUCTION

The symmetric, nonrelativistic harmonic-oscillator quark model has been shown by a number of people^{1,2} to offer considerable promise of helping to organize the wealth of data in the resonance region for high-energy phenomena. We intend here to bring some of these results together in a unified method of calculation in order to judge

ficing theoretical adequacy for simplicity. We shall choose a relativistic theory which is naive and obviously wrong in its simplicity, but which is definite and in which we can calculate as many things as possible – not expecting the results to agree exactly with experiment, but to see how closely our “shadow of the truth” equation gives a partial reflection of reality. In our attempt to maintain simplicity, we shall evidently have to

Feynman et al. acknowledge the existence of time separation between quarks inside the hadron

They choose not to do anything about it, because they do not know what to do.

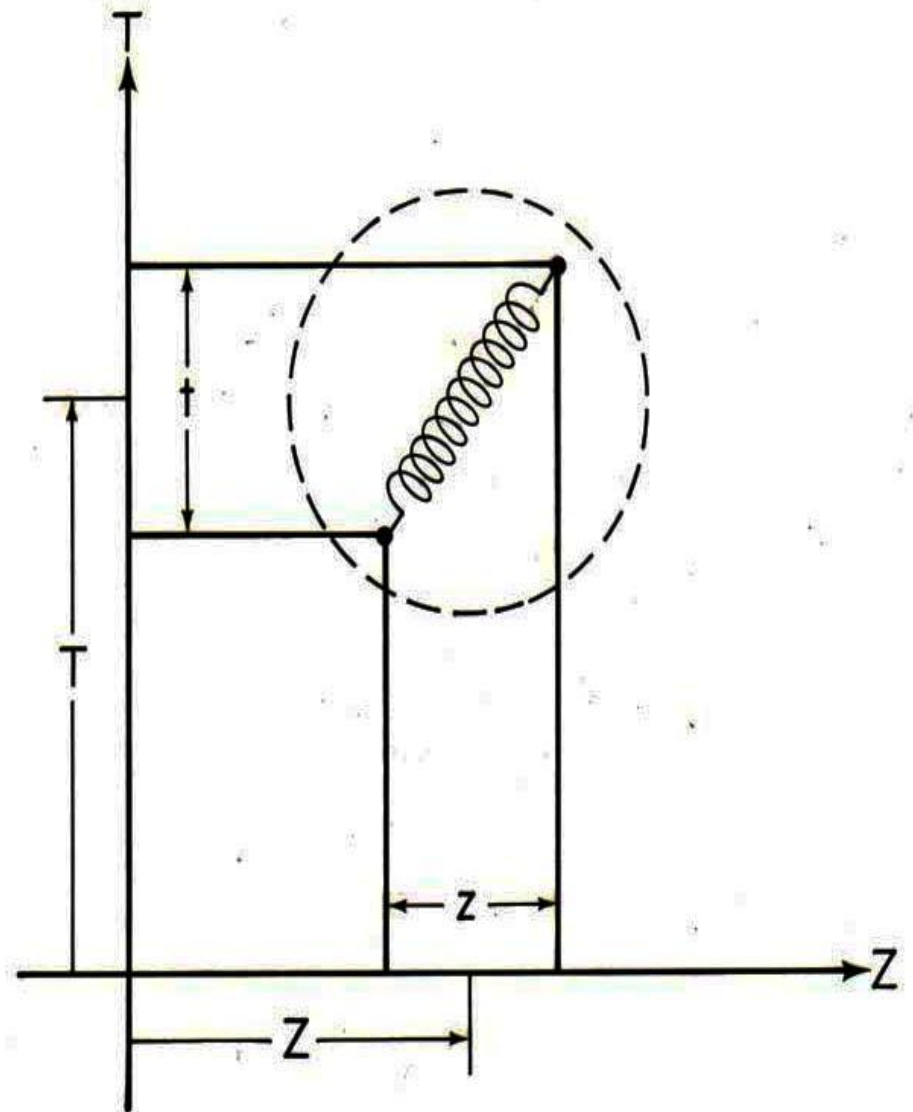
They fail to mention earlier works on this subject.

Main Issue

Time
Separation

Entanglement

Hidden
Variable



Yukawa (1953), Markov (1953),
Man'ko (1965), Fujimura (1970)



I am known as a person who
maintains webpages

<http://ysfine.com>

for conference catalog

<http://ysfine.com/princeton>

for beautiful campus

<http://ysfine.com/einstein>

for Einstein

I look very old these days.

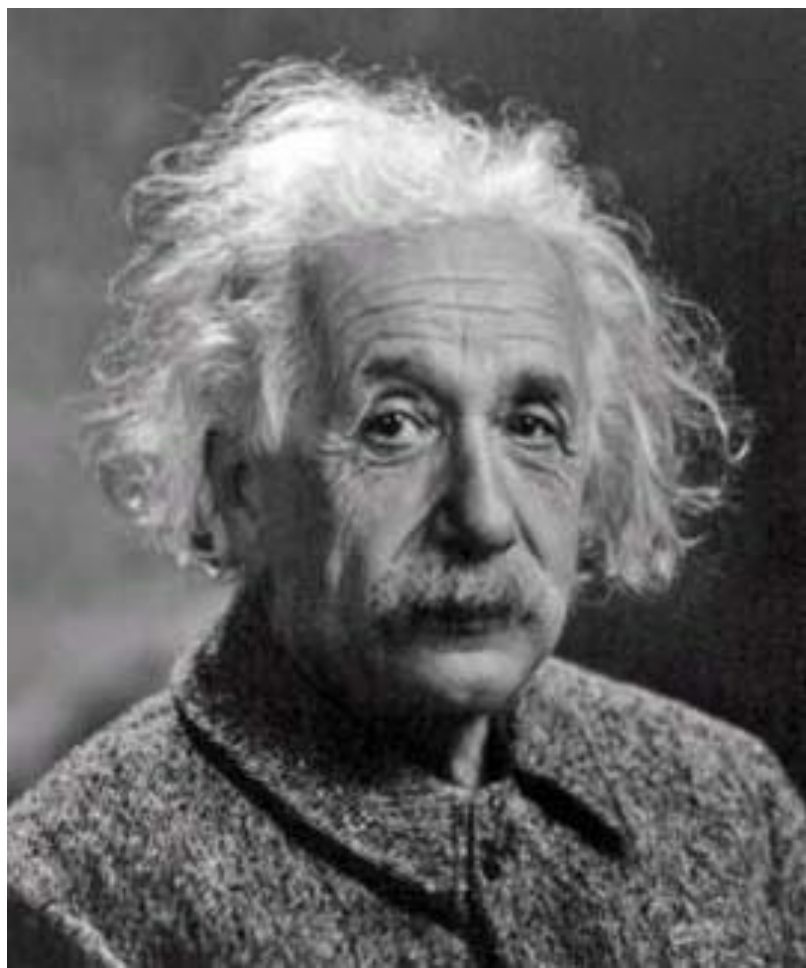
- Did I meet Einstein?
- No. I was too young
- I met some people who met him.



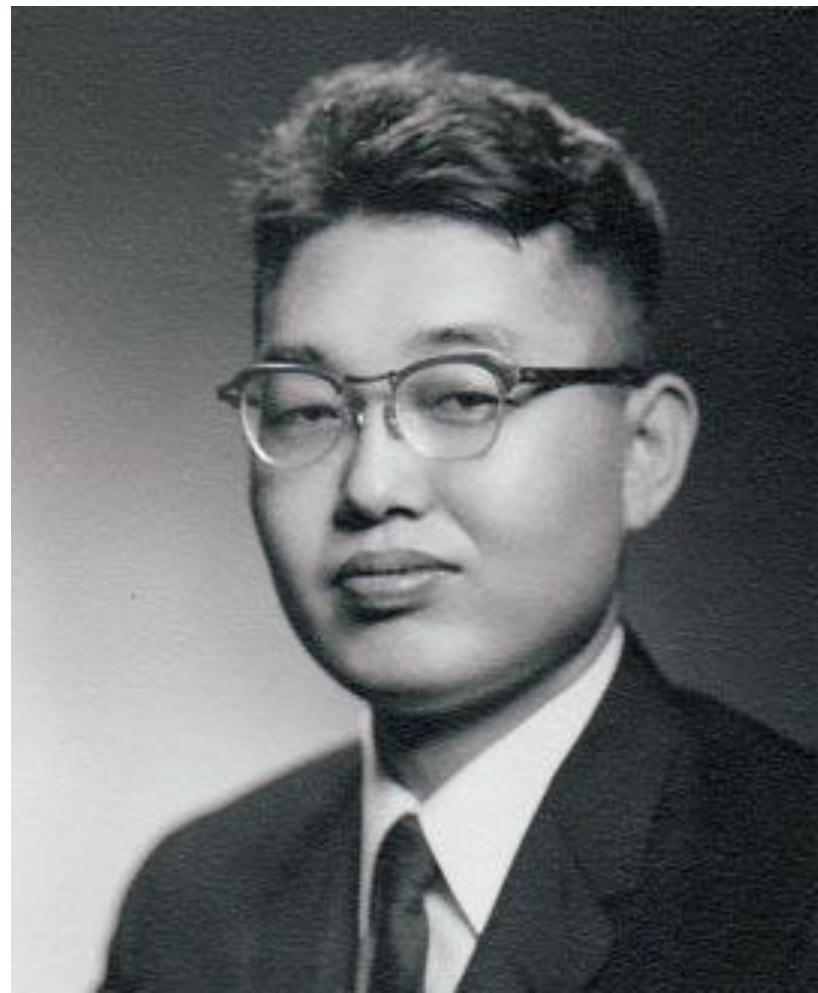
ORREN JACK TURNER
PRINCETON, N.J.

Orren Jack Turner

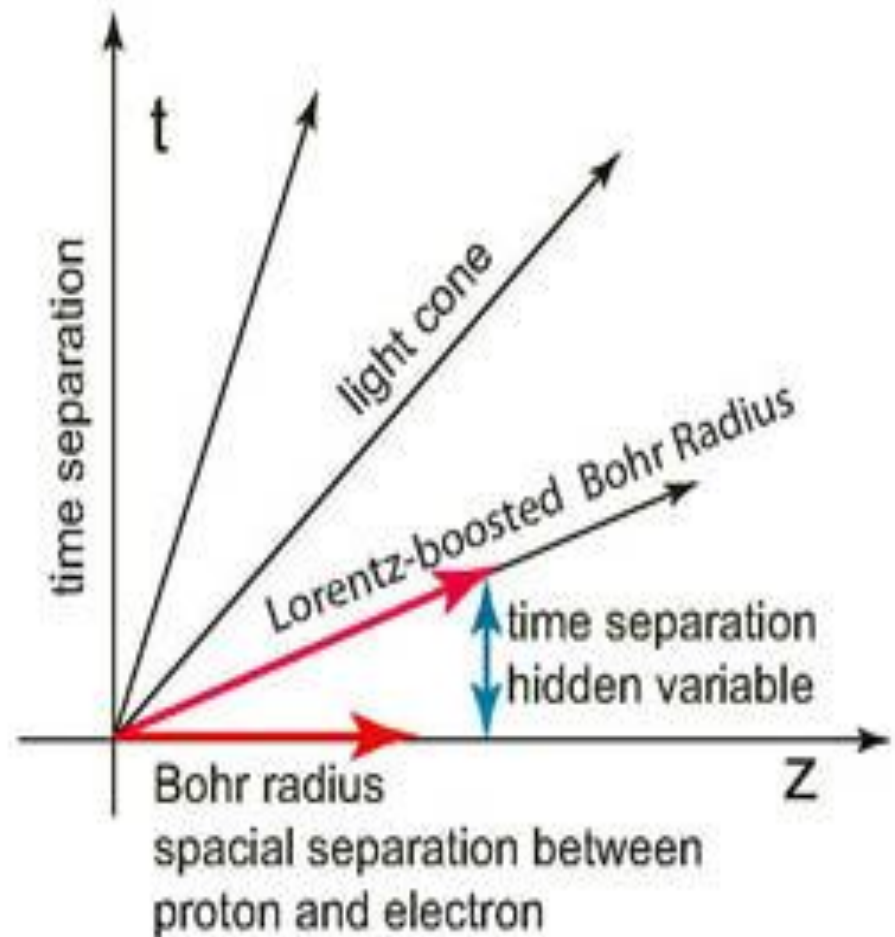
1947



1961



Hidden behind Einstein and Bohr



Niels Bohr

- **Bohr had a great respect for Einstein. Whenever he mentions “space,” he adds “time ‘ in his philosophical writings.**
- **However, the proton in the hydrogen atom is sitting at the center of the absolute frame.**

Einstein perhaps had a picture of Lorentz contraction, but never mentioned

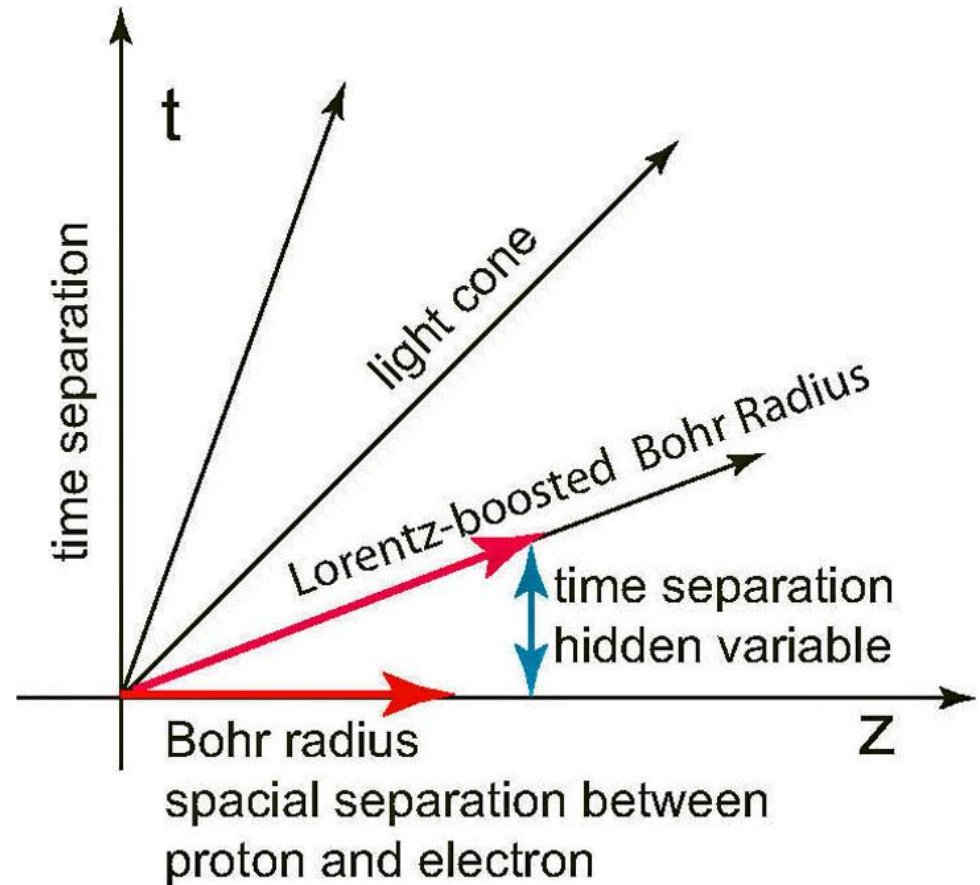
Bell's Picture of Lorentz Boost



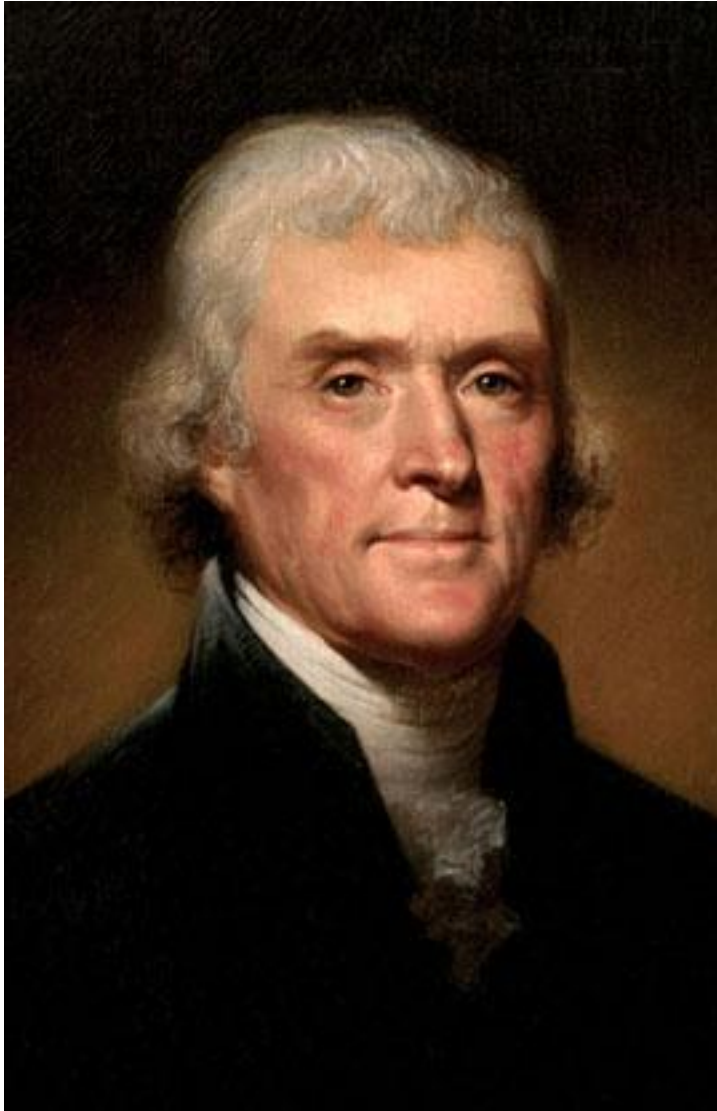
Longitudinal Axis

Bohr radius: space separation

- **Time separation was a hidden variable to them.**



Evolution of Environment



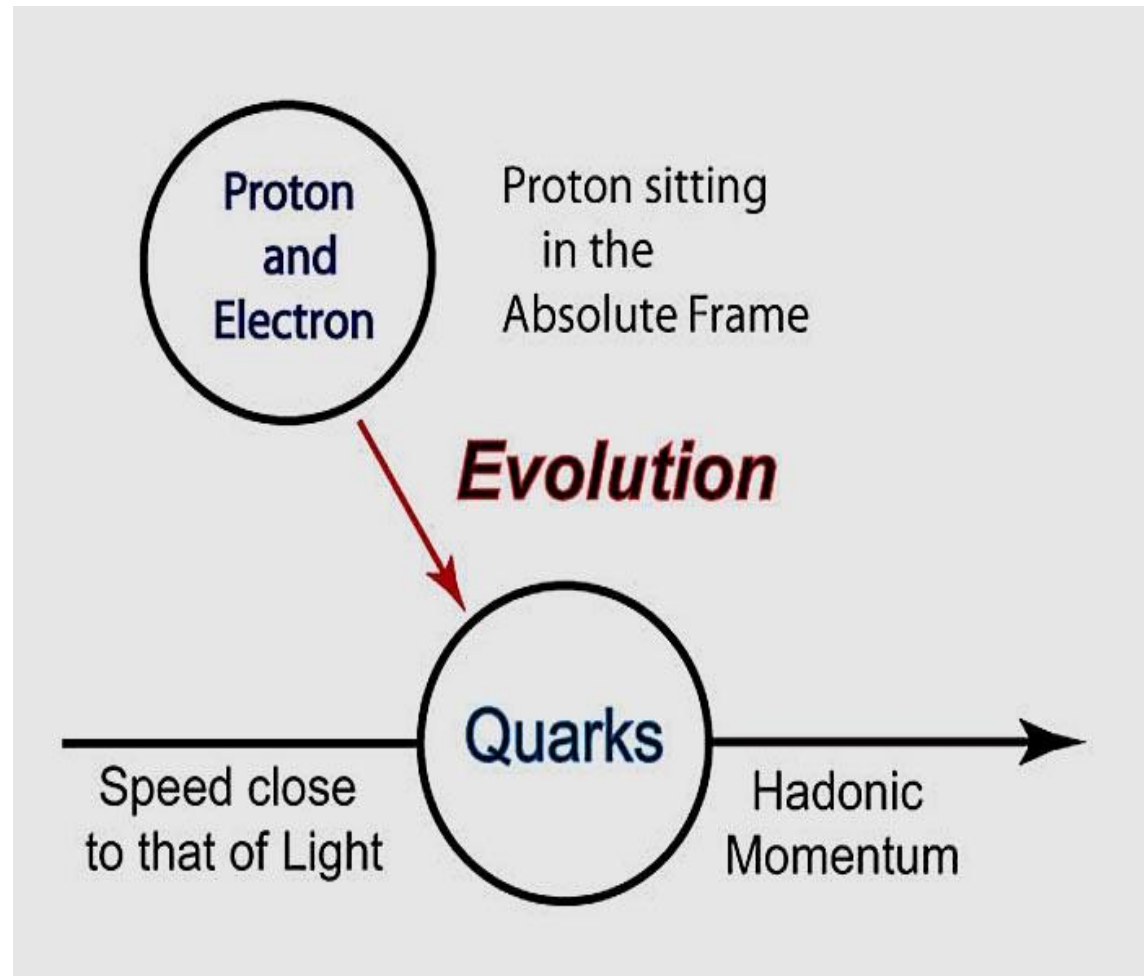
- All men are created equal.
- How about all women?
- How about his servants?

Jefferson's Monticello



Evolution of hydrogen atom

- Hydrogen becomes a hadron, a bound state of quarks.
- Speed of proton close to that of light.



**Hydrogen Atom becomes the
Hadron (bound state of two quarks)
in the Lorentz-covariant world.**

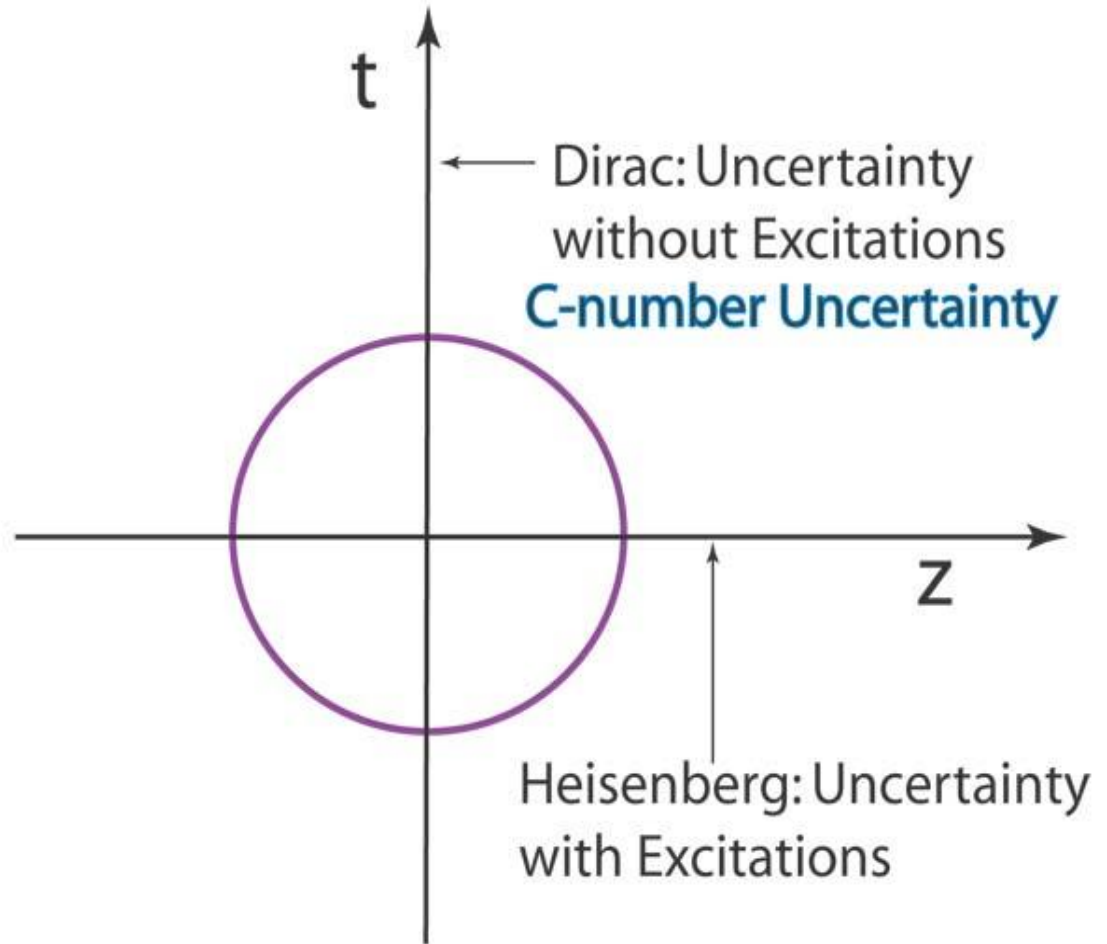
Running Waves



Running Waves

Time
separation
well defined.

Dirac 1927
and 1945

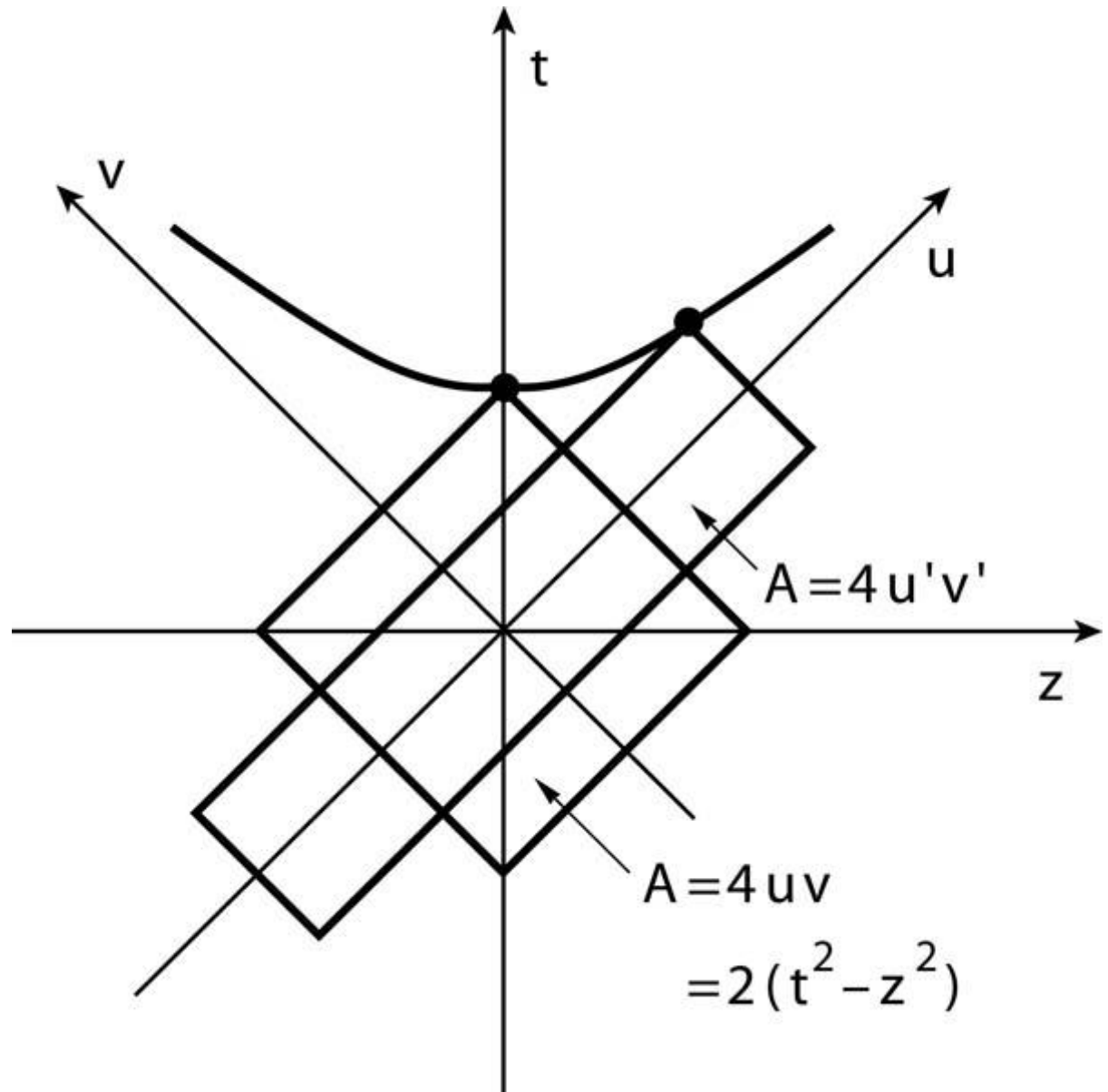


Space-time asymmetry? Dirac should have talked to Wigner



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Institute of Physics

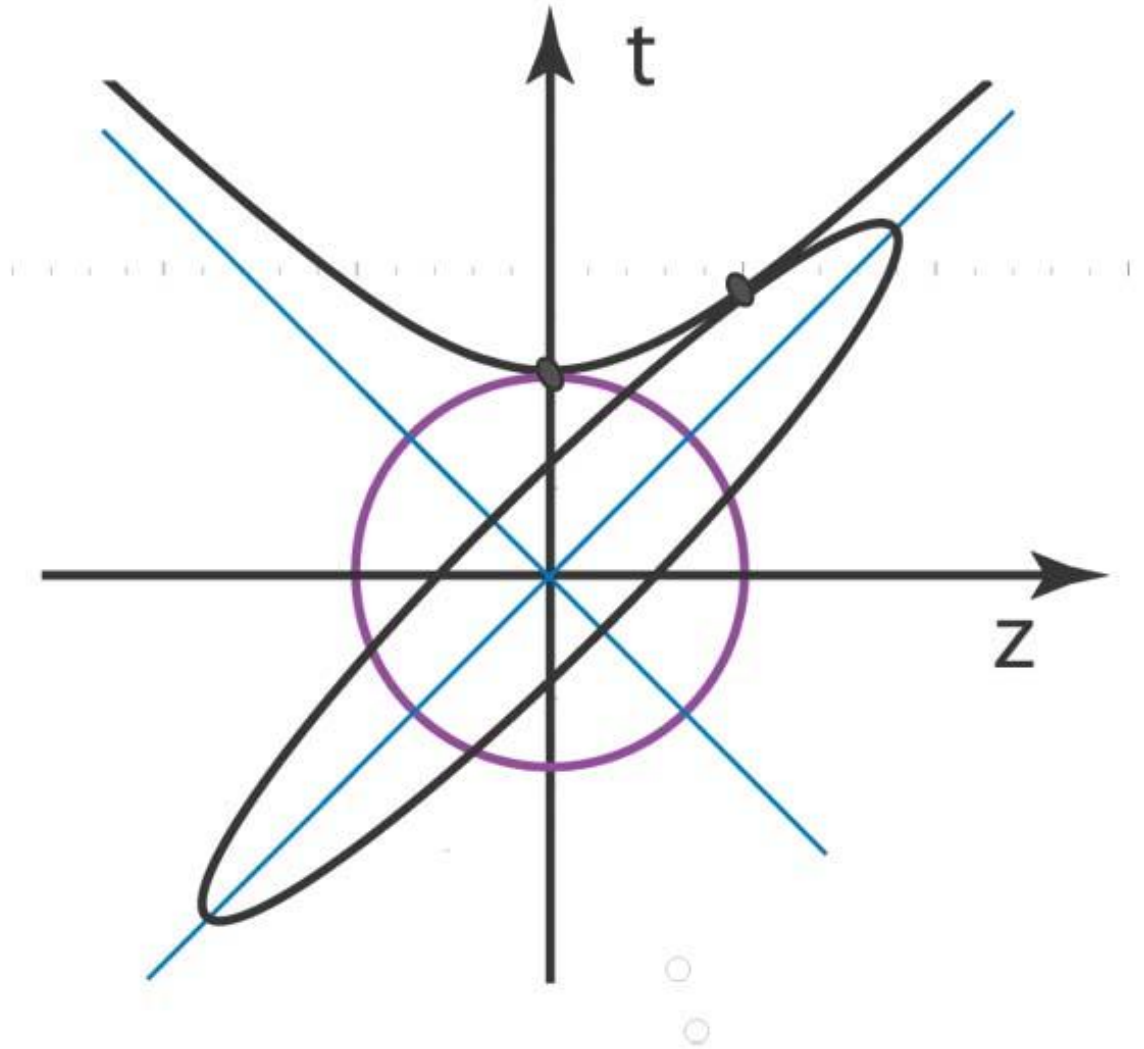
Dirac
1945



Physics

or

Science
Fiction

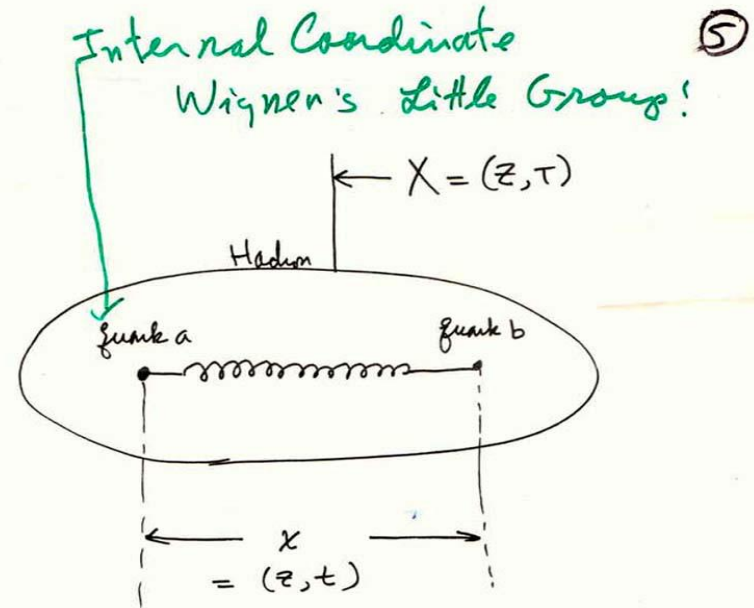


Let us invite Feynman



Covariant Bound States (Standing Waves)

- Bound States: Hydrogen Atom or Harmonic Oscillators.
- Feynman chooses osc. wave functions to understand the covariant world.
- Hadron consisting of two quarks. Overall coordinate and space-time separation.



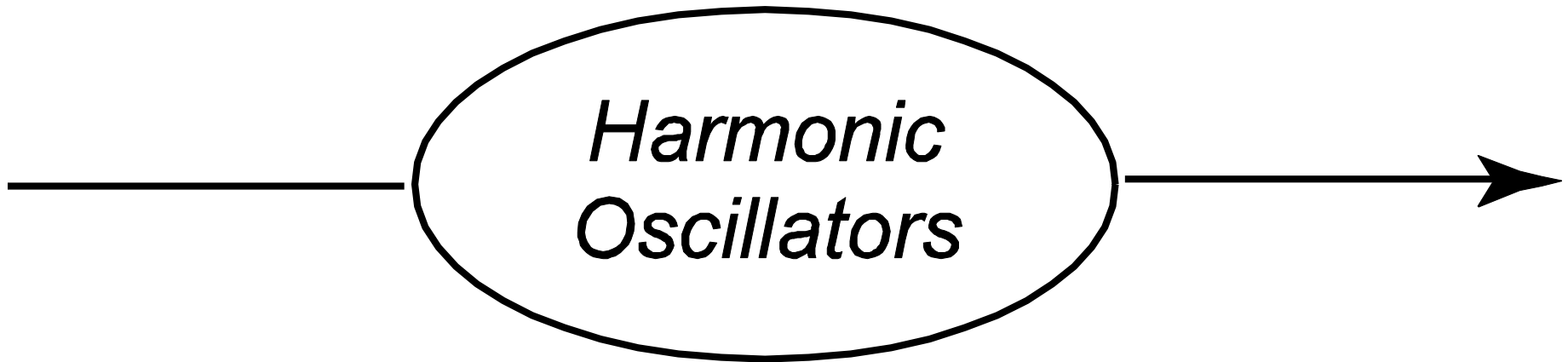
$$x_a = (z_a, t_a), \quad x_b = (z_b, t_b),$$

$$X = \frac{1}{2} (x_a + x_b),$$

$$x = \frac{1}{2\sqrt{2}} (x_b - x_a) \quad \leftarrow \text{Internal Space-Time Structure}$$

Feynman said

Feynman Diagrams

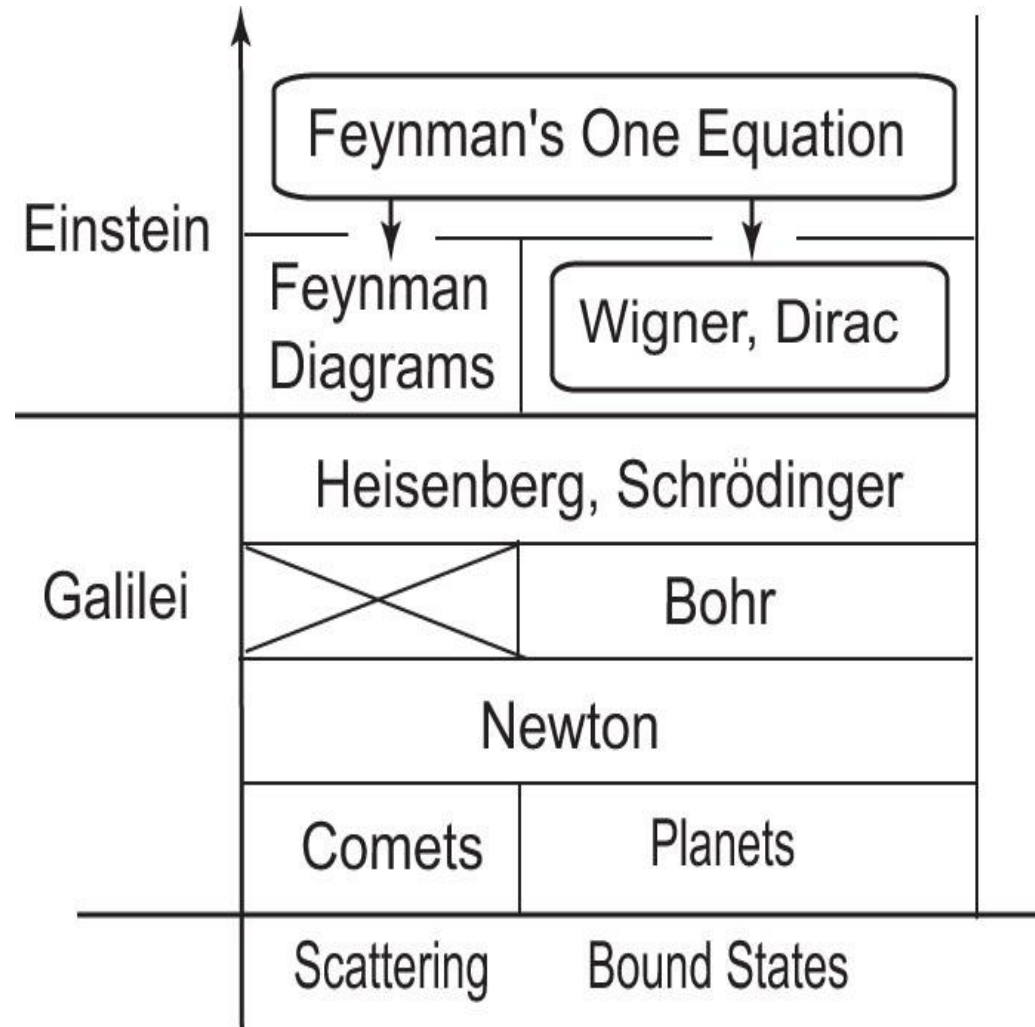


Feynman Diagrams

Feynman's One Equation

In his 1971 paper, Feynman wrote down one partial differential equation to cover both comets and planets.

Where is the Squeezed State?



Feynman's One Equation

Let us start with the Lorentz-invariant differential equation which Feynman wrote down with his co-workers. In order to explain the hadron as a bound state of the quarks, they start with two quarks whose space-time coordinates are x_μ^a and x_μ^b respectively. They then wrote down the equation

$$\left\{ -\frac{1}{2} \left[\left(\frac{\partial}{\partial x_\mu^a} \right)^2 + \left(\frac{\partial}{\partial x_\mu^b} \right)^2 \right] + \frac{1}{16} (x_\mu^a - x_\mu^b)^2 + m_0^2 \right\} \phi(x_\mu^a, x_\mu^b) = 0, \quad (1)$$

If we use the hadronic and the quark separation coordinates as

$$\begin{aligned} X_\mu &= \frac{1}{2} (x_\mu^a + x_\mu^b), \\ x_\mu &= \frac{1}{2\sqrt{2}} (x_\mu^a - x_\mu^b), \end{aligned} \quad (2)$$

respectively, it is possible to consider the separation of the variables:

$$\phi(x_\mu^a, x_\mu^b) = f(X_\mu) \psi(x_\mu) \quad (3)$$

then the differential equation can be separated into the following two equations.

$$\left\{ \left(\frac{\partial}{\partial X_\mu} \right)^2 + m_0^2 + (\lambda + 1) \right\} f(X_\mu) = 0 \quad (4)$$

for the hadronic coordinate, and

$$\frac{1}{2} \left\{ - \left(\frac{\partial}{\partial x_\mu} \right)^2 + x_\mu^2 \right\} \psi(x_\mu) = (\lambda + 1) \psi(x_\mu), \quad (5)$$

for the coordinate of quarks inside the hadron.

The first differential equation of a Klein-Gordon for the hadronic coordinate. The Klein-Gordon equation is Lorentz-invariant and is the starting point for quantum field theory for scattering processes with Feynman diagrams. In the present case, the solution takes the form

$$f(X) = \exp(\pm i P \cdot X), \quad (6)$$

with

$$-P^2 = m_0^2 + (\lambda + 1). \quad (7)$$

Thus, the hadronic mass is determined from m_0 and λ . The λ parameter is determined from Eq.(5) for internal space-time coordinate.

Hadrons – Running waves

Quarks inside – Harmonic Oscillator

The differential equation of Eq.(5) is also Lorentz-invariant equation, but its solution can take different forms depending on the separable coordinate systems with their boundary conditions. The problem is to choose the set of solutions which can tell us physics properly based on the existing rules of quantum mechanics and relativity.

If we ignore time-like variable in Eq.(5) is the Schrödinger-type equation for a harmonic oscillator. The problem is the existence of the time-like variable. If we ignore it, it is the equation for non-relativistic quantum mechanics, but it is not Lorentz-invariant. If we include it, we have to give a physical interpretation to this variable.

Feynman's Oscillators

Let us use the simplest hadron consisting of two quarks bound together with an attractive force, and consider their space-time positions x_a and x_b , and use the variables

$$X = (x_a + x_b)/2, \quad x = (x_a - x_b)/2\sqrt{2}. \quad (9)$$

The four-vector X specifies where the hadron is located in space and time, while the variable x measures the space-time separation between the quarks.

In their classic 1971 paper [9], Feynman, Kislinger and Ravndal start with Lorentz-invariant differential equation.

$$\frac{1}{2} \left\{ x_\mu^2 - \frac{\partial^2}{\partial x_\mu^2} \right\} \psi(x) = \lambda \psi(x). \quad (10)$$

Although this paper contained the above mentioned original idea of Feynman, it contains some serious mathematical flaws. Feynman *et al.* start with a Lorentz-invariant differential equation for the harmonic oscillator for the quarks

Feynman was successful in solving the mysteries of Regge poles, but totally failed in providing Lorentz-covariant solutions of his own equation for harmonic oscillators.

From the Lorentzian point of view, his paper is total mess.

Dirac and Feynman in Poland (1962)

**When Dirac told me
American physicists
should spend more
time on Lorentz
covariance, he was
talking about
Feynman.**



When Dirac was talking to me about American physicists in 1962, he was talking about Feynman.

Thus, one way to fix Feynman's problem was to import Dirac's work.

bound together inside a hadron. For the two-quark system, they write the wave function of the form

$$\exp \left\{ -\frac{1}{2} (z^2 - t^2) \right\}, \quad (11)$$

where z and t are the longitudinal and time-like separations between the quarks. This form is invariant under the boost, but is not normalizable in the t variable. We do not know what physical interpretation to give to this the above expression.

Is it possible to fix Feynman's Oscillators?

If we adopt Dirac's picture which allows excitations along the longitudinal coordinate and forbid excitations along the

Yukawa (1953), Markov (1953),
Man'ko (1965), Fujimura (1970)



time coordinate, the wave function takes the form

$$\psi_0^n(z, t) = C_n H_n(z) \exp \left\{ -\frac{1}{2} (z^2 + t^2) \right\}, \quad (12)$$

If the system is boosted along the z direction, the z and t variables in the above wave function should be replaced by z' and t' respectively with

$$z' = (\cosh \eta)z - (\sinh \eta)t, \quad t' = (\cosh \eta)t - (\sinh \eta)z. \quad (13)$$

The Lorentz-boosted wave function takes the form

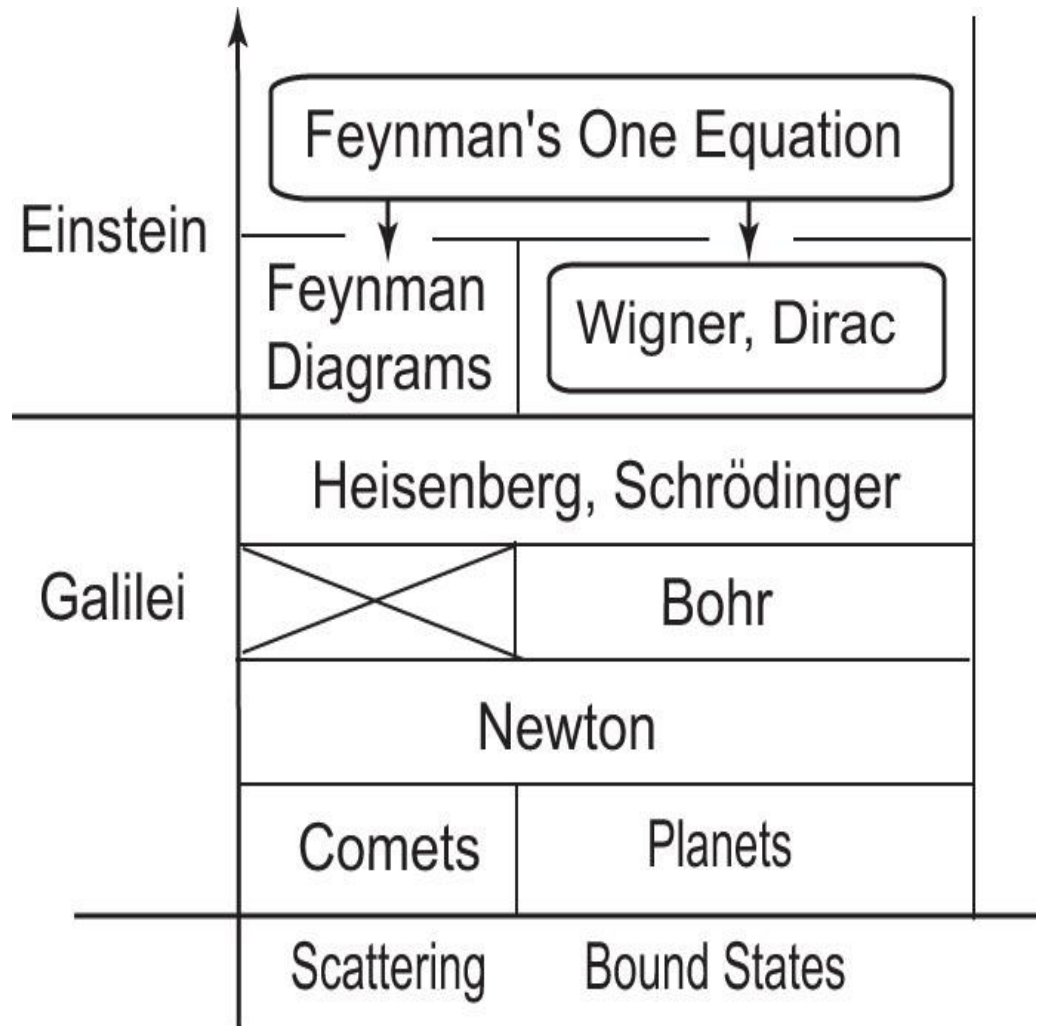
$$\psi_\eta^n(z, t) = H_n(z') \exp \left\{ -\frac{1}{2} (z'^2 + t'^2) \right\}, \quad (14)$$

We are interested in space-time localizations of the wave function dictated by the Gaussian factor. In Dirac's light-cone coordinate system,

$$u = \frac{z + t}{\sqrt{2}}, \quad u = \frac{z - t}{\sqrt{2}}, \quad (15)$$

Feynman's One Equation

In order to handle
the differential
equation
Feynman
wrote down, we
have to import
things from both
Wigner and Dirac.



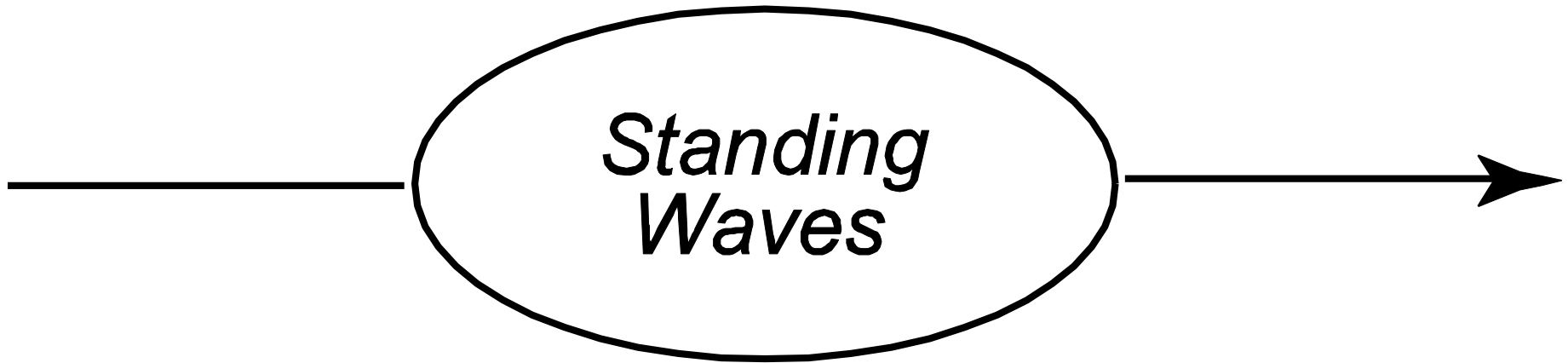
Two famous brothers in law



Scanned at the American
Institute of Physics

Running waves and Standing Waves

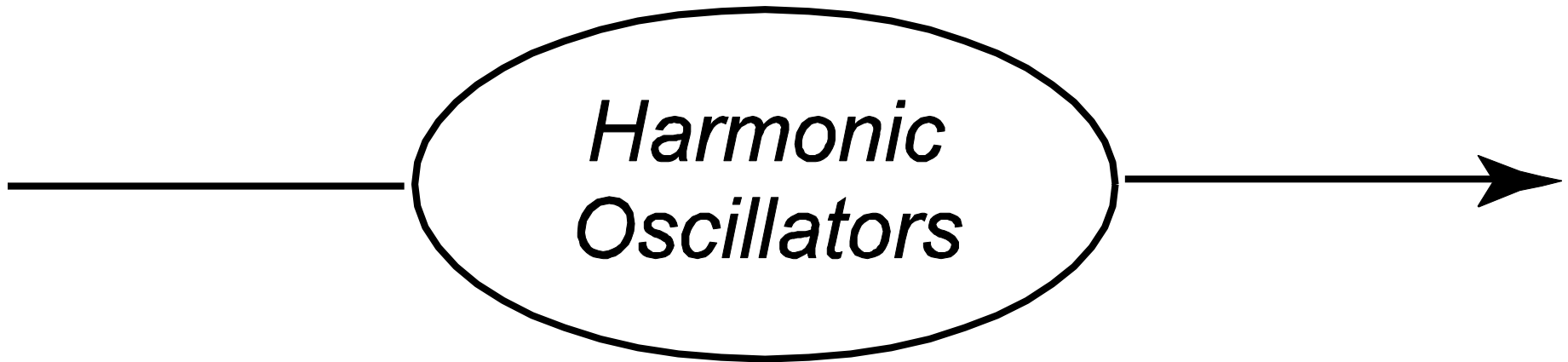
Running Waves



Running Waves

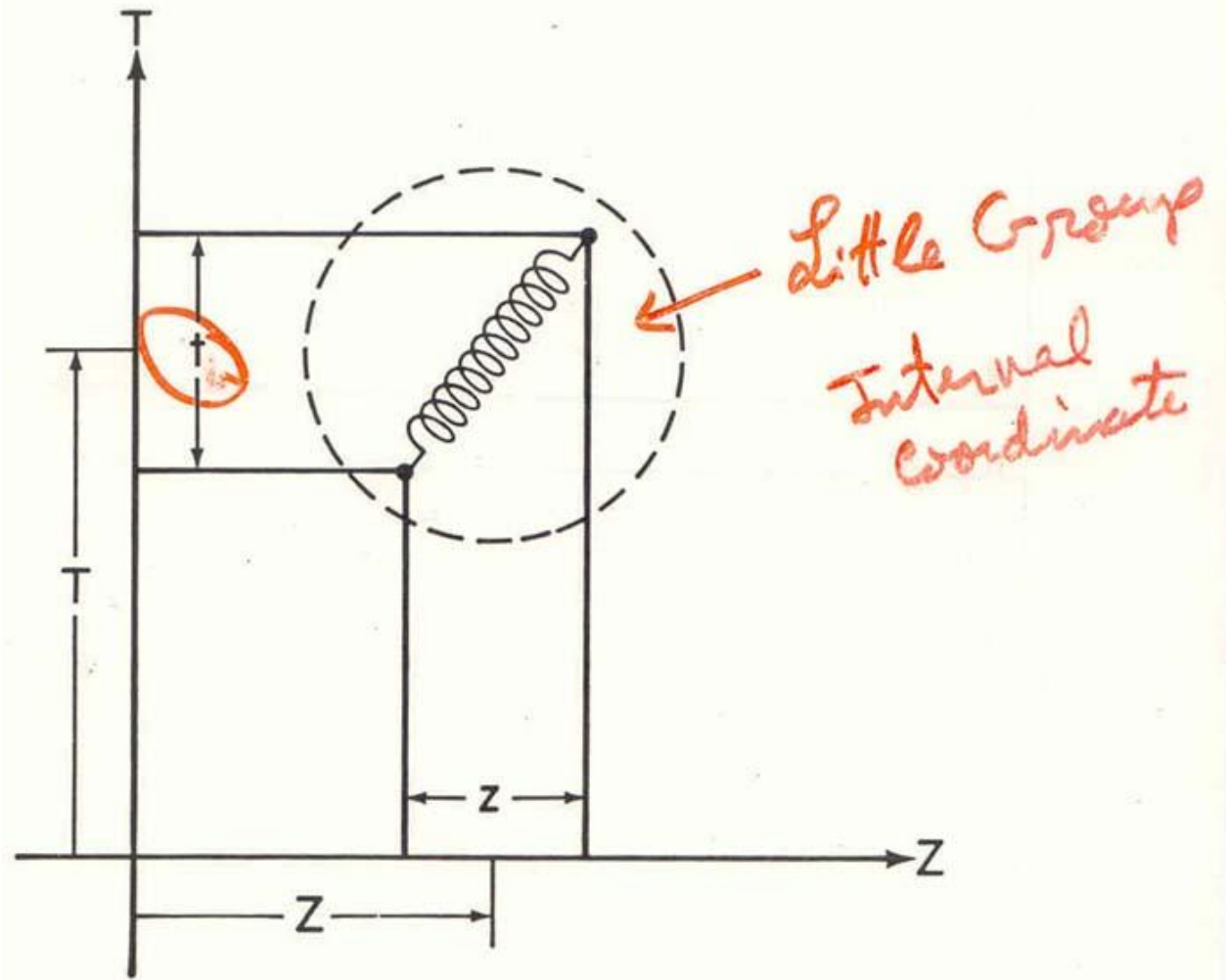
Feynman said

Feynman Diagrams

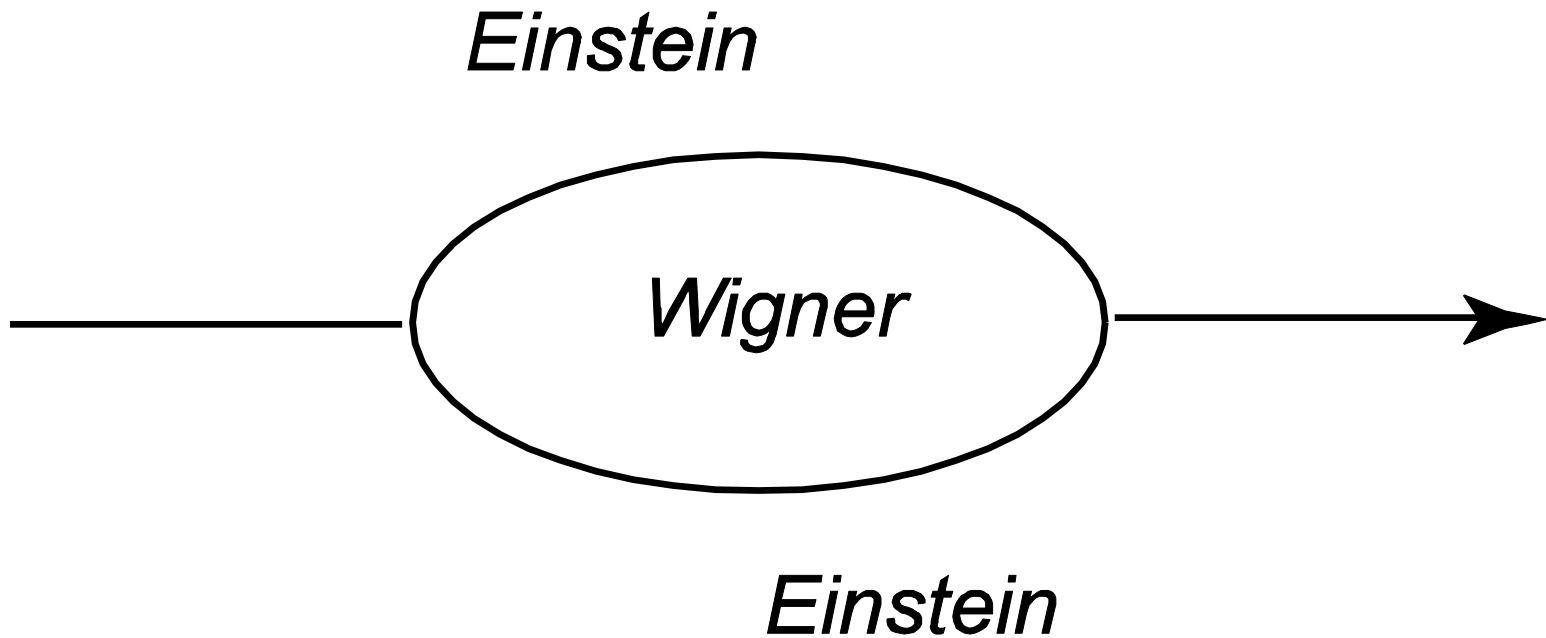


Feynman Diagrams

Time
separation



Wigner's Little Group



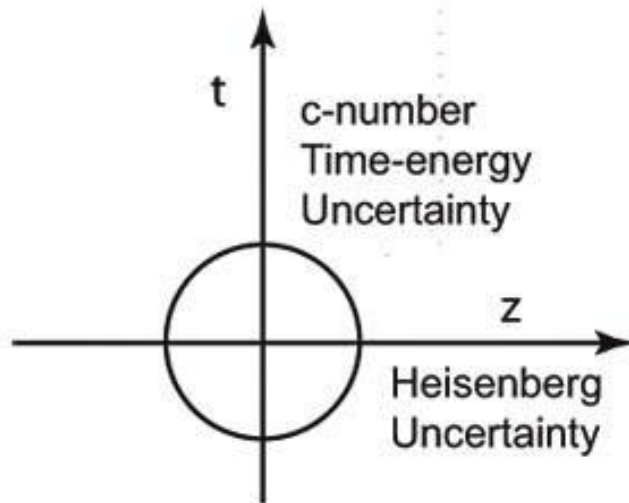
- The space-time symmetry inside the hadron is smaller than the Lorentz group. It is isomorphic to $O(3)$, three-dimensional rotation group.

After fixing up Feynman's oscillators

**We can assert that Dirac's Five Books
are for the real world.**

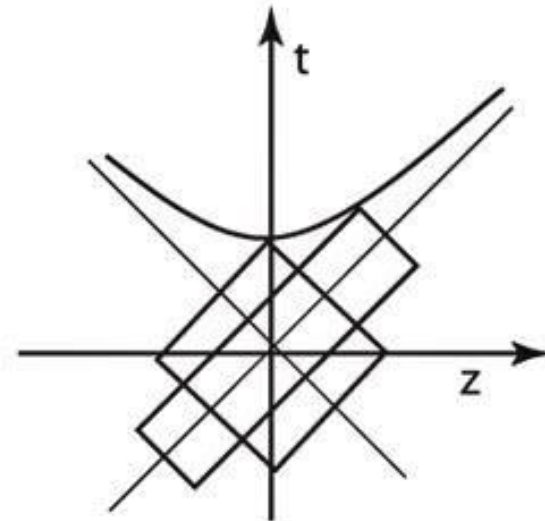
Dirac (1927)

Quantum Mechanics

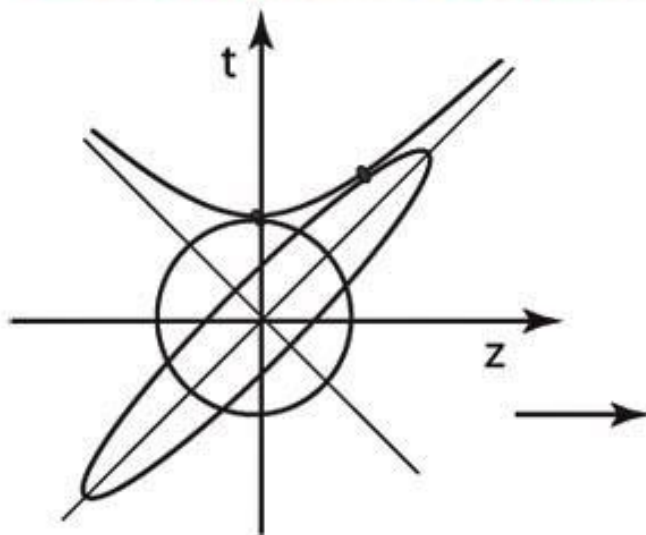


Dirac (1949)

Lorentz Covariance



Lorentz-covariant Quantum Mechanics



Feynman

1969: Parton Picture

1971: Oscillator Model for
Bound States

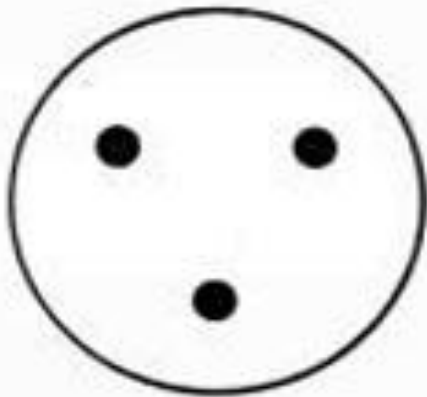
1973: Rest of the Universe

Parton Picture

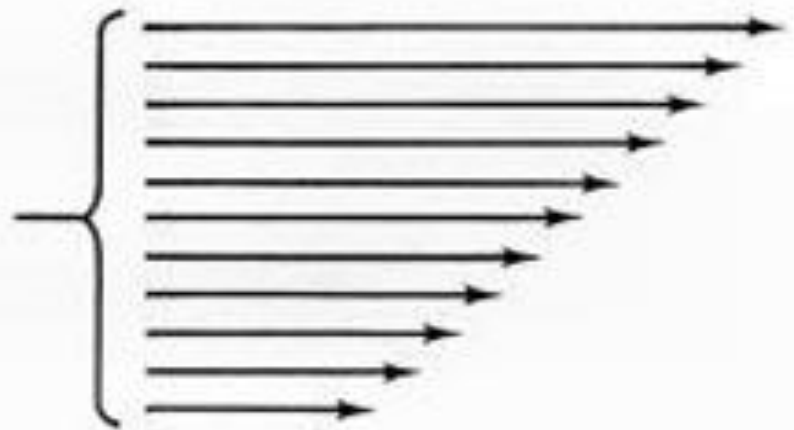
Coherence
to
Incoherence



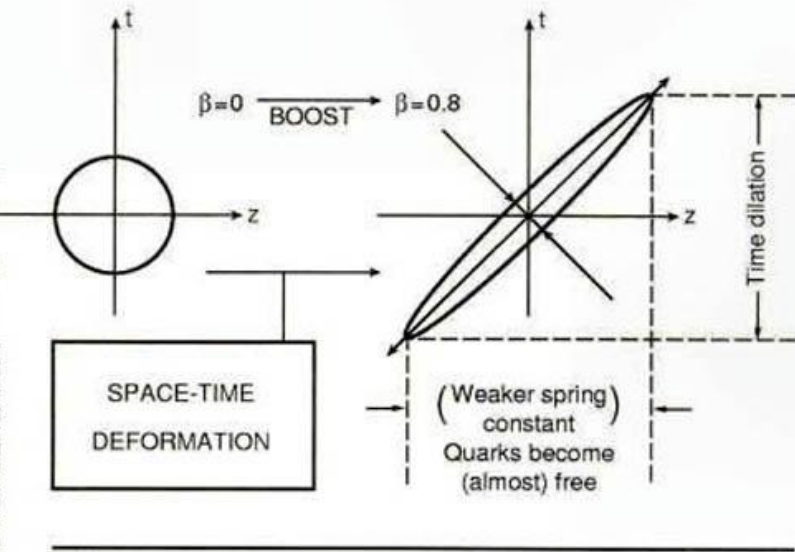
QUARKS



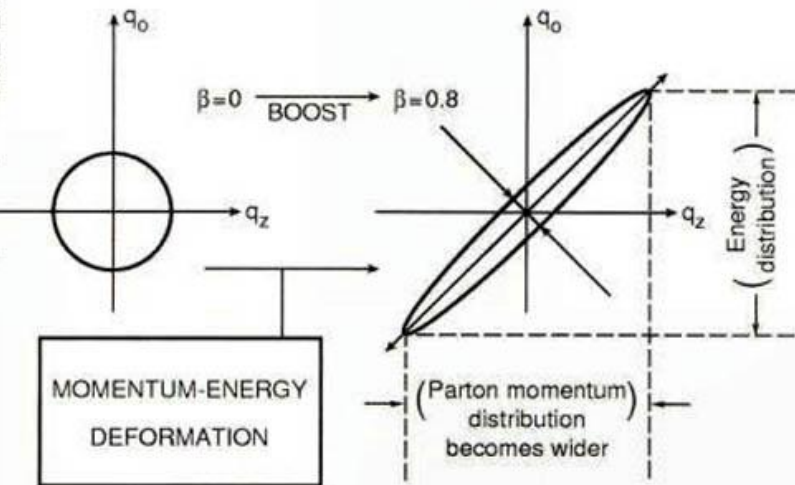
PARTONS



QUARKS $\longrightarrow$ PARTONS



Lorentz-covariant
Picture of Feynman's
Parton Model



	Massive Slow	← between →	Massless Fast
Energy Momentum	$E = \frac{p^2}{2m}$	$E = \sqrt{m^2 + p^2}$	$E = p$
Spin, Gauge Helicity	S_3 S_1 S_2	← Little Groups →	S_3 Gauge Trans.
Quarks Partons	Quark Model	← (Covariant Phase Space) →	Parton Model

Two different aspects
(quarks and partons) of nature
coming from one thing!

Observable Gauge Transformations in the Parton Picture

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(Received 6 February 1989)

The internal space-time symmetry of rapidly moving composite particles is studied in terms of the little groups of the Poincaré group. It is shown that Feynman's x in his parton picture is a gauge-transformation parameter.

PACS numbers: 11.30.Cp, 03.30.+p, 12.40.Aa

The internal space-time symmetries of elementary particles are governed by the little groups of the Poincaré group.¹ The little groups for massive and massless particles are locally isomorphic to $O(3)$ and $E(2)$ (two-dimensional Euclidean group), respectively. It was shown recently that the $E(2)$ -like little group for massless particles is an infinite-momentum, zero-mass limit of the $O(3)$ -like little group.^{2,3} The role of the little groups is illustrated in the second row of Fig. 1.

The purpose of this Letter is to show that the internal space-time symmetry of composite particles can be formulated within the framework of Wigner's little groups, and that Feynman's x parameter in his parton picture⁴ is a gauge-transformation parameter. This will unify the second and third rows of Fig. 1. This figure is from the recent paper by Kim and Wigner which deals primarily with the third row.⁵

Wigner's little group is the maximal subgroup of the Lorentz group whose transformations leave the four-momentum of a given particle invariant.¹ For a massive

point particle, there is a Lorentz frame in which the particle is at rest. In this frame, the little group is the three-dimensional rotation group. This is the fundamental symmetry associated with the concept of spin.

The internal space-time symmetry of massless particles is governed by the cylindrical group which is locally isomorphic to $E(2)$.³ In this case, we can visualize a circular cylinder whose axis is parallel to the momentum. On the surface of this cylinder, we can rotate a point around the axis or translate along the direction of the axis. The rotational degree of freedom is associated with the helicity, while the translation corresponds to a gauge transformation in the case of photons.³

This translational degree of freedom is shared by all massless particles, including neutrinos and gravitons.⁶ Indeed, the requirement of invariance under this symmetry leads to the polarization of neutrinos.^{6,7} Since this translational degree of freedom is a gauge degree of freedom for photons, we can extend the concept of gauge transformations to all massless particles⁸ and massive particles in the infinite-momentum limit.

It is not difficult to associate the symmetry of a point particle with that of a composite particle if they are massive and at rest, because both of them are governed by the three-dimensional rotation group.⁹ The story is quite different for rapidly moving composite particles or hadrons. Does a rapidly moving hadron have the same set of space-time degrees of freedom as that of photons? We can study this problem by constructing a cylindrical symmetry for a hadron with infinite momentum. Then, is this symmetry consistent with Feynman's parton picture? This is the question we would like to address in the present paper.

The group of Lorentz transformations is generated by three rotation and three boost generators.^{1,6,7} If we use the four-vector convention $x^\mu = (x, y, z, t)$, the generators of rotations around and boosts along the z axis take the form

$$J_3 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \end{pmatrix}. \quad (1)$$

FIG. 1. Slow and fast particles. Einstein's $E = (p^2 + m^2)^{1/2}$ unifies the energy-momentum relations for massive (nonrelativistic) particles and for massless particles. The second row indicates that the little group of the Poincaré group unifies the internal space-time symmetries of massive and massless particles, as is discussed in Ref. 2. The third row states that the covariant phase-space picture of quantum mechanics forms the physical basis for the covariant harmonic-oscillator formalism which has been shown to give a unified picture of the quark model and the parton picture.

	Massive Slow	between	Massless Fast
Energy Momentum	$E = \frac{p^2}{2m}$	$E = \sqrt{m^2 + p^2}$	$E = p$
Spin, Gauge Helicity	S_3 S_1 S_2	Little Groups	S_3 Gauge Trans.
Quarks Partons	Gell-Mann's Quark Model	(Covariant Phase Space)	Feynman's Parton Model

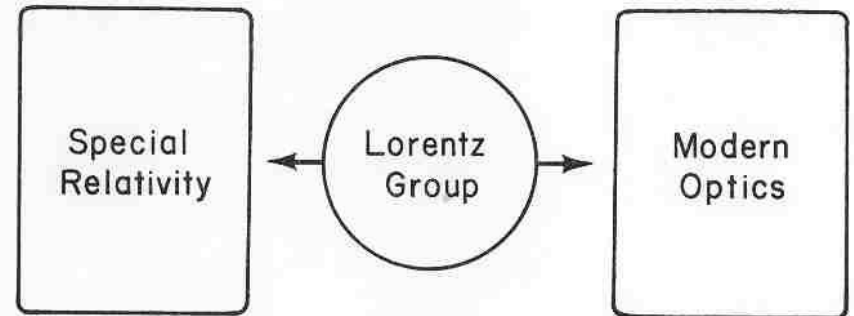
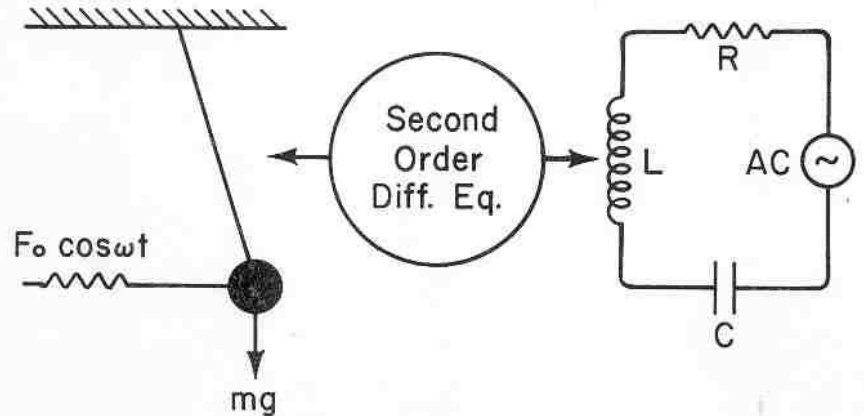
Register
ownership
with
authorities

Phys. Rev.
Letters

**Still uncomfortable with
the time separation?**

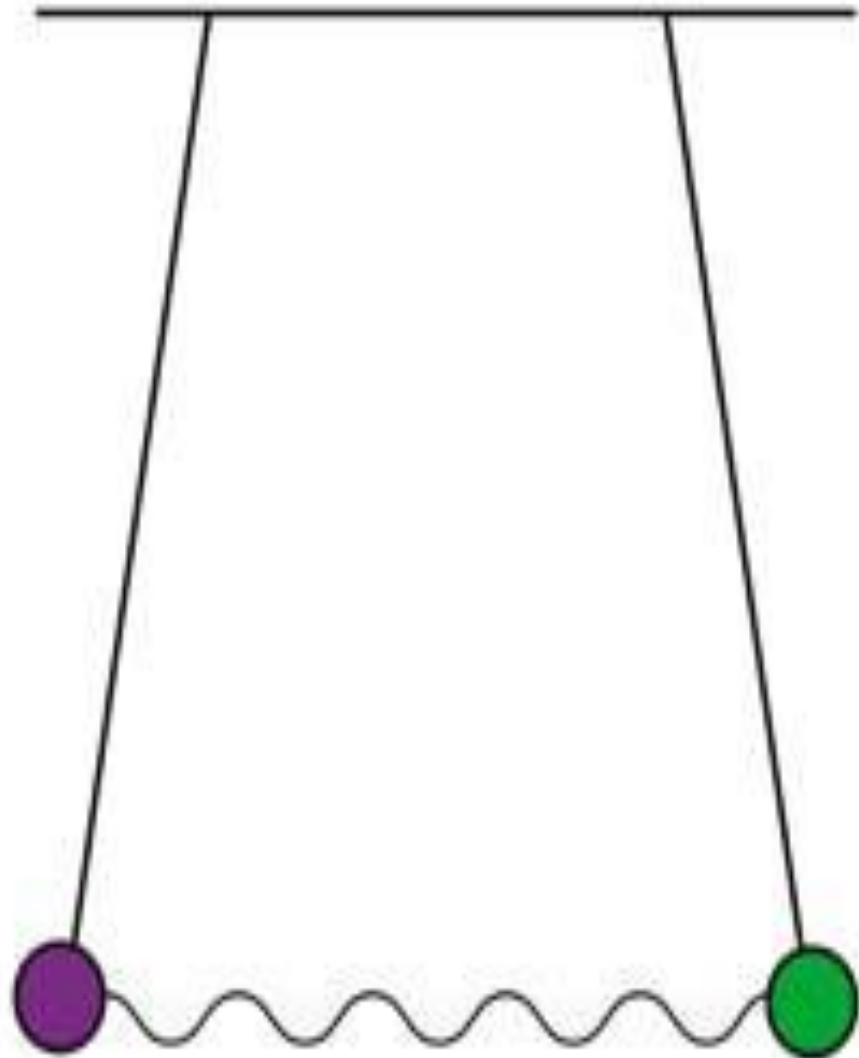
Same math

Physics can
be
transferred



Coupled Harmonic Oscillators

Theory of
Everything
in Physics



Coupled Oscillators

$$H = \frac{1}{2} \left\{ \frac{1}{m} p_1^2 + \frac{1}{m} p_2^2 + A x_1^2 + A x_2^2 + 2C x_1 x_2 \right\}. \quad (1)$$

If we choose coordinate variables

$$y_1 = \frac{1}{\sqrt{2}} (x_1 + x_2), \quad y_2 = \frac{1}{\sqrt{2}} (x_1 - x_2), \quad (2)$$

the Hamiltonian can be written as

$$H = \frac{1}{2m} \{p_1^2 + p_2^2\} + \frac{K}{2} \{e^{-2\eta} y_1^2 + e^{2\eta} y_2^2\}, \quad (3)$$

where

$$K = \sqrt{A^2 - C^2}, \quad \exp(2\eta) = \sqrt{\frac{A - C}{A + C}}, \quad (4)$$

If y_1 and y_2 are measured in units of $(mK)^{1/4}$, the ground-state wave function of this oscillator system is

$$\psi_\eta(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{1}{2} (e^{-\eta} y_1^2 + e^{\eta} y_2^2) \right\}, \quad (5)$$

Dirac
1962

Two-
oscillator
system

Two-mode
squeezed
states

$$\hat{L}_1 = \frac{1}{2} (a_1^\dagger a_2 + a_2^\dagger a_1), \quad \hat{L}_2 = \frac{1}{2i} (a_1^\dagger a_2 - a_2^\dagger a_1),$$

$$\hat{L}_3 = \frac{1}{2} (a_1^\dagger a_1 - a_2^\dagger a_2), \quad \hat{S}_3 = \frac{1}{2} (a_1^\dagger a_1 + a_2^\dagger a_2),$$

$$\hat{K}_1 = -\frac{1}{4} (a_1^\dagger a_1^\dagger + a_1 a_1 - a_2^\dagger a_2^\dagger - a_2 a_2),$$

$$\hat{K}_2 = \frac{i}{4} (a_1^\dagger a_1^\dagger - a_1 a_1 + a_2^\dagger a_2^\dagger - a_2 a_2),$$

$$\hat{K}_3 = \frac{1}{2} (a_1^\dagger a_2^\dagger + a_1 a_2),$$

$$\hat{Q}_1 = -\frac{i}{4} (a_1^\dagger a_1^\dagger - a_1 a_1 - a_2^\dagger a_2^\dagger + a_2 a_2),$$

$$\hat{Q}_2 = -\frac{1}{4} (a_1^\dagger a_1^\dagger + a_1 a_1 + a_2^\dagger a_2^\dagger + a_2 a_2),$$

$$\hat{Q}_3 = \frac{i}{2} (a_1^\dagger a_2^\dagger - a_1 a_2).$$

Let us write the wave function of Eq.(5) in terms of x_1 and x_2 , then

$$\psi_{\eta}(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{1}{4} \left[e^{-\eta} (x_1 + x_2)^2 + e^{\eta} (x_1 - x_2)^2 \right] \right\}. \quad (6)$$

When the system is decoupled with $\eta = 0$, this wave function becomes

$$\psi_0(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{1}{2} (x_1^2 + x_2^2) \right\}. \quad (7)$$

$$\psi_{\eta}(x_1, x_2) = \frac{1}{\cosh \eta} \sum_k \left(\tanh \frac{\eta}{2} \right)^k \phi_k(x_1) \phi_k(x_2), \quad (8)$$

where $\phi_k(x)$ is the harmonic oscillator wave function for the k -th excited state. This expansion serves as the mathematical basis for squeezed states of light in quantum optics [3], among other applications.

with the Lorentz boost of the form

$$u' = e^{\eta}u, \quad v' = e^{-\eta}v, \quad (16)$$

the Lorentz-boosted wave function becomes

$$\psi_{\eta}(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp \left\{ -\frac{1}{2} \left(e^{-2\eta}u^2 + e^{2\eta}v^2 \right) \right\}, \quad (17)$$

This wave function can be written as

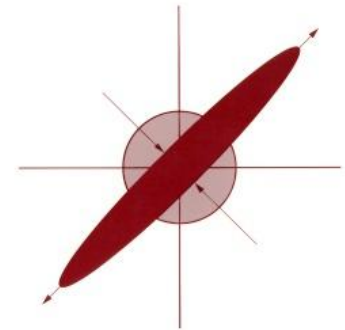
$$\psi_{\eta}(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp \left\{ -\frac{1}{4} \left[e^{-2\eta}(z+t)^2 + e^{2\eta}(z-t)^2 \right] \right\}. \quad (18)$$

This form is mathematically identical to that of the coupled oscillators. The physics of x_1 and x_2 can be translated into that of x and t , namely the longitudinal and time-like separations.

The same formula leads to

$$\psi_{\eta}(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{1}{4} \left[e^{-\eta} (x_1 + x_2)^2 + e^{\eta} (x_1 - x_2)^2 \right] \right\}.$$

Top: Squeezed State



Bottom: Entangled State

$$\psi_{\eta}(x_1, x_2) = \frac{1}{\cosh \eta} \sum_k \left(\tanh \frac{\eta}{2} \right)^k \phi_k(x_1) \phi_k(x_2)$$

The adventure of our science of physics is a perpetual attempt to recognize that the different aspects of nature are really different aspects of the same thing.



Feynman was a Kantianist,
so was Einstein.

Entropy Problem

- First, let us look at what he says about the density matrix. In his 1973 book, Feynman says

2.1 INTRODUCTION TO DENSITY MATRICES

When we solve a quantum-mechanical problem, what we really do is divide the universe into two parts—the system in which we are interested and the rest of the universe. We then usually act as if the system in which we are interested comprised the entire universe. To motivate the use of density matrices, let us see what happens when we include the part of the universe outside the system.

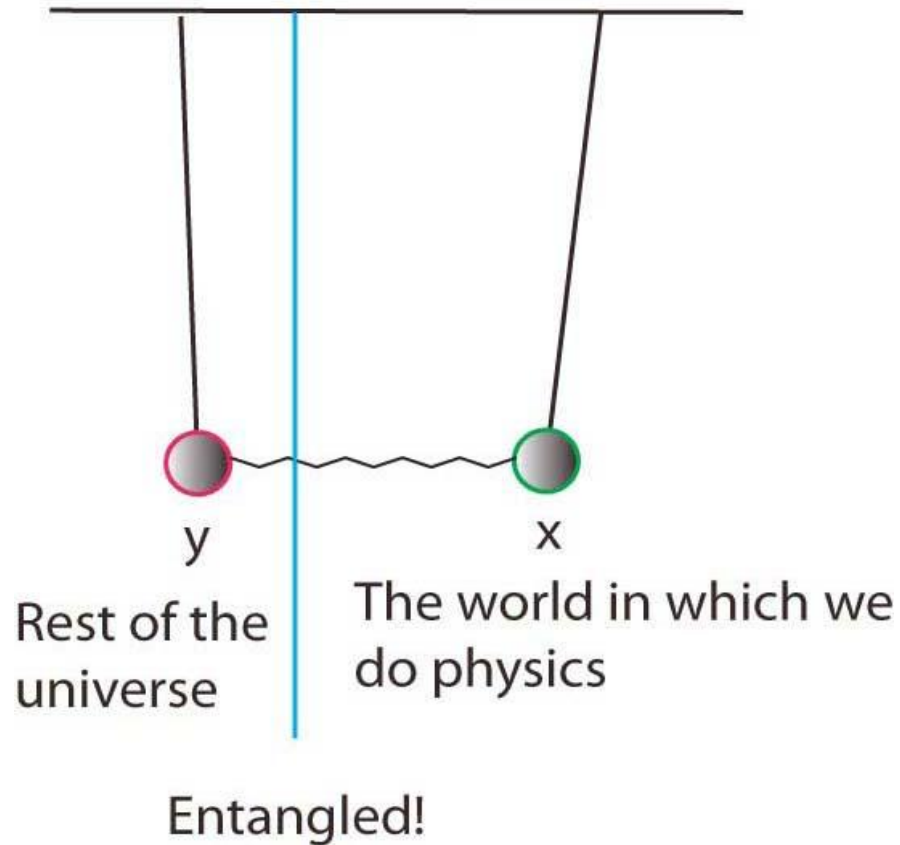
Let x describe the coordinates of the system, and let y describe the rest of the universe. Let $\varphi_i(x)$ be a complete set of wave functions. The most general wave function can be written

$$\psi(x, y) = \sum_i C_i(y) \varphi_i(x). \quad (2.1)$$

Here again, Feynman talks like a philosopher

- Here again, we can use harmonic oscillators to see what he really had in mind.
- In his original paper he uses a single oscillator.
- We should use coupled oscillators to decode what he was saying.

Rest of the universe -- Entanglement



- X variable: we do physics with.
- Y variable: we do not observe. – rest of the universe.

Two worlds are entangled.

$$\psi_{\eta}(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{1}{4} \left[e^{\eta} (x_1 - x_2)^2 + e^{-\eta} (x_1 + x_2)^2 \right] \right\}.$$

As was discussed in the literature for several different purposes, this wave function can be expanded as

$$\psi_{\eta}(x_1, x_2) = \frac{1}{\cosh \eta} \sum_k \left(\tanh \frac{\eta}{2} \right)^k \phi_k(x_1) \phi_k(x_2).$$

From this wave function, we can construct the pure-state density matrix

$$\rho(x_1, x_2; x'_1, x'_2) = \psi_{\eta}(x_1, x_2) \psi_{\eta}(x'_1, x'_2),$$

$$\rho(x, x') = \left(\frac{1}{\pi \cosh \eta} \right)^{1/2} \times \exp \left\{ - \left[\frac{(x + x')^2 + (x - x')^2 \cosh^2 \eta}{4 \cosh \eta} \right] \right\}.$$

The diagonal elements of the above density matrix are

$$\rho(x, x) = \left(\frac{1}{\pi \cosh \eta} \right)^{1/2} \exp \left(-x^2 / \cosh \eta \right).$$

With this expression, we can confirm the property of the density matrix: $Tr(\rho) = 1$. As for the trace of

which satisfies the condition $\rho^2 = \rho$:

$$\rho(x_1, x_2; x'_1, x'_2) = \int \rho(x_1, x_2; x''_1, x''_2) \rho(x''_1, x''_2; x'_1, x'_2) dx''_1 dx''_2.$$

If we are not able to make observations on the x_2 , we should take the trace of the ρ matrix with respect to the x_2 variable. Then the resulting density matrix is

$$\rho(x, x') = \int \psi_\eta(x, x_2) \{\psi_\eta(x', x_2)\}^* dx_2.$$

Here again, we have replaced x_1 and x'_1 by x and x' respectively. If we complete the integration over the x_2 variable,

ρ^2 , we can perform the integration

$$Tr(\rho^2) = \int \rho(x, x') \rho(x', x) dx' dx = \frac{1}{\cosh \eta},$$

which is less than one for nonzero values of η .

The density matrix can also be calculated from the expansion of the wave function given in Eq.(.). If we perform the integral of Eq.(.), the result is

$$\rho(x, x') = \left(\frac{1}{\cosh(\eta/2)} \right)^2 \sum_k \left(\tanh \frac{\eta}{2} \right)^{2k} \phi_k(x) \phi_k^*(x'),$$

which leads to $Tr(\rho) = 1$. It is also straightforward to compute the integral for $Tr(\rho^2)$. The calculation

leads to

$$Tr(\rho^2) = \left(\frac{1}{\cosh(\eta/2)} \right)^4 \sum_k \left(\tanh \frac{\eta}{2} \right)^{4k}.$$

The sum of this series is $(1/\cosh \eta)$, which is the same as the result of Eq.().

This is of course due to the fact that we are averaging over the x_2 variable which we do not measure. The standard way to measure this ignorance is to calculate the entropy defined as

$$S = -Tr(\rho \ln(\rho)),$$

where S is measured in units of Boltzmann's constant. If we use the density matrix given in Eq.(),

the entropy becomes

$$S = 2 \left\{ \cosh^2 \left(\frac{\eta}{2} \right) \ln \left(\cosh \frac{\eta}{2} \right) - \sinh^2 \left(\frac{\eta}{2} \right) \ln \left(\sinh \frac{\eta}{2} \right) \right\}.$$

This expression can be translated into a more familiar form if we use the notation

$$\tanh \frac{\eta}{2} = \exp \left(- \frac{\hbar \omega}{kT} \right),$$

where ω is given in Eq.(??). The ratio $\hbar \omega / kT$ is a dimensionless variable. In terms of this variable, the entropy takes the form

$$S = \left(\frac{\hbar \omega}{kT} \right) \frac{1}{\exp(\hbar \omega / kT) - 1} - \ln [1 - \exp(-\hbar \omega / kT)].$$

which satisfies the condition $\rho^2 = \rho$:

$$\rho(x_1, x_2; x'_1, x'_2) = \int \rho(x_1, x_2; x''_1, x''_2) \rho(x''_1, x''_2; x'_1, x'_2) dx''_1 dx''_2.$$

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The density matrix can also be calculated from the expansion of the wave function given in Eq.(.). If we perform the integral of Eq.(.), the result is

$$\rho(x, x') = \left(\frac{1}{\cosh(\eta/2)} \right)^2 \sum_k \left(\tanh \frac{\eta}{2} \right)^{2k} \phi_k(x) \phi_k^*(x'),$$

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where ω is given in Eq.(??). The ratio $\hbar \omega / kT$ is a dimensionless variable. In terms of this variable, the entropy takes the form

$$S = \left(\frac{\hbar \omega}{kT} \right) \frac{1}{\exp(\hbar \omega / kT) - 1} - \ln [1 - \exp(-\hbar \omega / kT)].$$

Back to Space-time physics

- Time separation variable between the quarks exists, but they are not observed.
- It indeed belongs to Feynman's rest of the universe.
- First photon becomes z
- Second photon becomes t .

Observable Gauge Transformations in the Parton Picture

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The internal space-time symmetry of rapidly moving composite particles is studied in terms of the little groups of the Poincaré group. It is shown that Feynman's x in his parton picture is a gauge-transformation parameter.

PACS numbers: 11.30.Cp, 03.30.+p, 12.40.Aa

The internal space-time symmetries of elementary particles are governed by the little groups of the Poincaré group.¹ The little groups for massive and massless particles are locally isomorphic to $O(3)$ and $E(2)$ (two-dimensional Euclidean group), respectively. It was shown recently that the $E(2)$ -like little group for massless particles is an infinite-momentum, zero-mass limit of the $O(3)$ -like little group.^{2,3} The role of the little groups is illustrated in the second row of Fig. 1.

The purpose of this Letter is to show that the internal space-time symmetry of composite particles can be formulated within the framework of Wigner's little groups, and that Feynman's x parameter in his parton picture⁴ is a gauge-transformation parameter. This will unify the second and third rows of Fig. 1. This figure is from the recent paper by Kim and Wigner which deals primarily with the third row.⁵

Wigner's little group is the maximal subgroup of the Lorentz group whose transformations leave the four-momentum of a given particle invariant.¹ For a massive

point particle, there is a Lorentz frame in which the particle is at rest. In this frame, the little group is the three-dimensional rotation group. This is the fundamental symmetry associated with the concept of spin.

The internal space-time symmetry of massless particles is governed by the cylindrical group which is locally isomorphic to $E(2)$.³ In this case, we can visualize a circular cylinder whose axis is parallel to the momentum. On the surface of this cylinder, we can rotate a point around the axis or translate along the direction of the axis. The rotational degree of freedom is associated with the helicity, while the translation corresponds to a gauge transformation in the case of photons.³

This translational degree of freedom is shared by all massless particles, including neutrinos and gravitons.⁶ Indeed, the requirement of invariance under this symmetry leads to the polarization of neutrinos.^{6,7} Since this translational degree of freedom is a gauge degree of freedom for photons, we can extend the concept of gauge transformations to all massless particles⁸ and massive particles in the infinite-momentum limit.

It is not difficult to associate the symmetry of a point particle with that of a composite particle if they are massive and at rest, because both of them are governed by the three-dimensional rotation group.⁹ The story is quite different for rapidly moving composite particles or hadrons. Does a rapidly moving hadron have the same set of space-time degrees of freedom as that of photons? We can study this problem by constructing a cylindrical symmetry for a hadron with infinite momentum. Then, is this symmetry consistent with Feynman's parton picture? This is the question we would like to address in the present paper.

The group of Lorentz transformations is generated by three rotation and three boost generators.^{1,6,7} If we use the four-vector convention $x^\mu = (x, y, z, t)$, the generators of rotations around and boosts along the z axis take the form

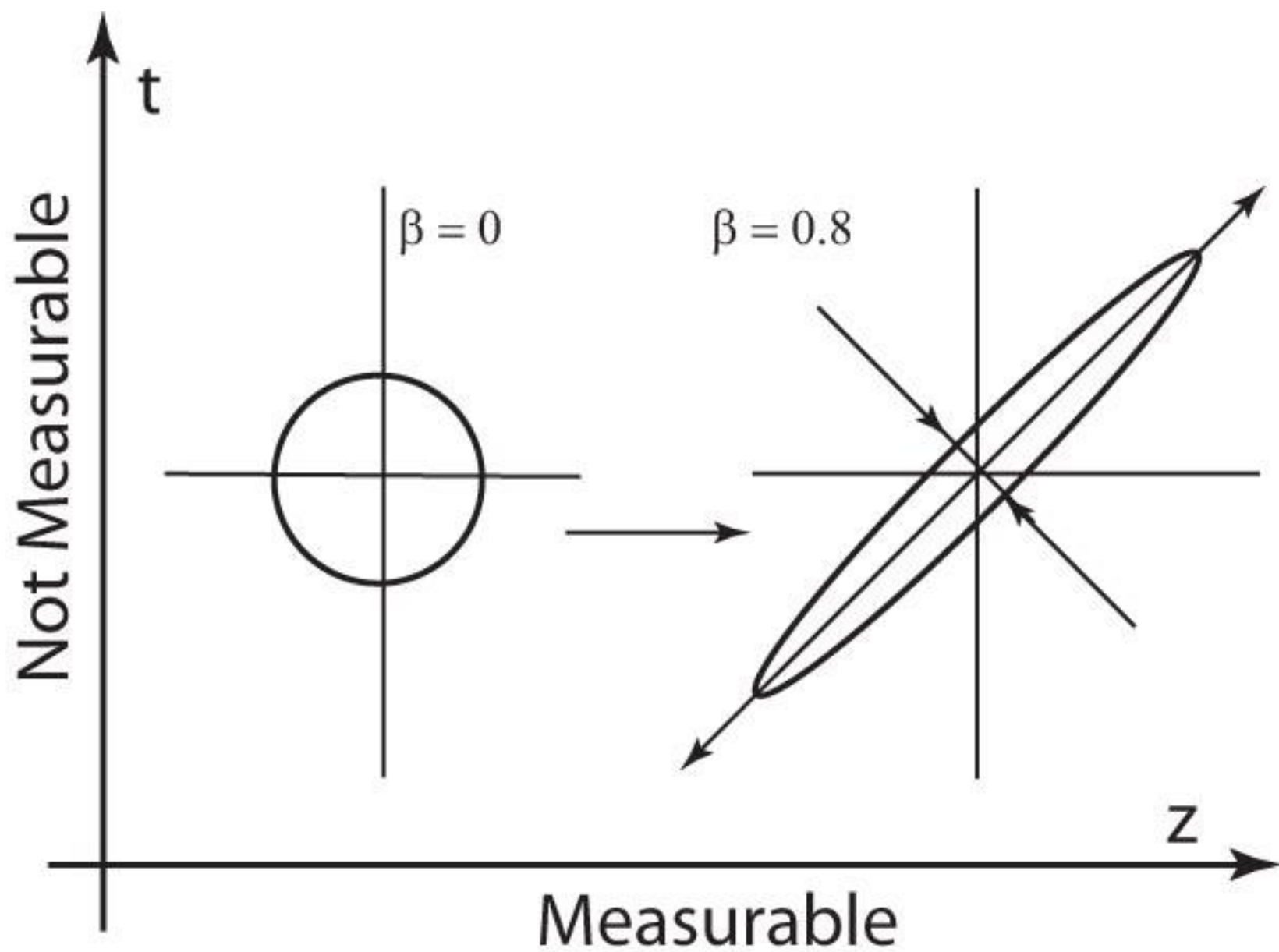
$$J_3 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \end{pmatrix}. \quad (1)$$

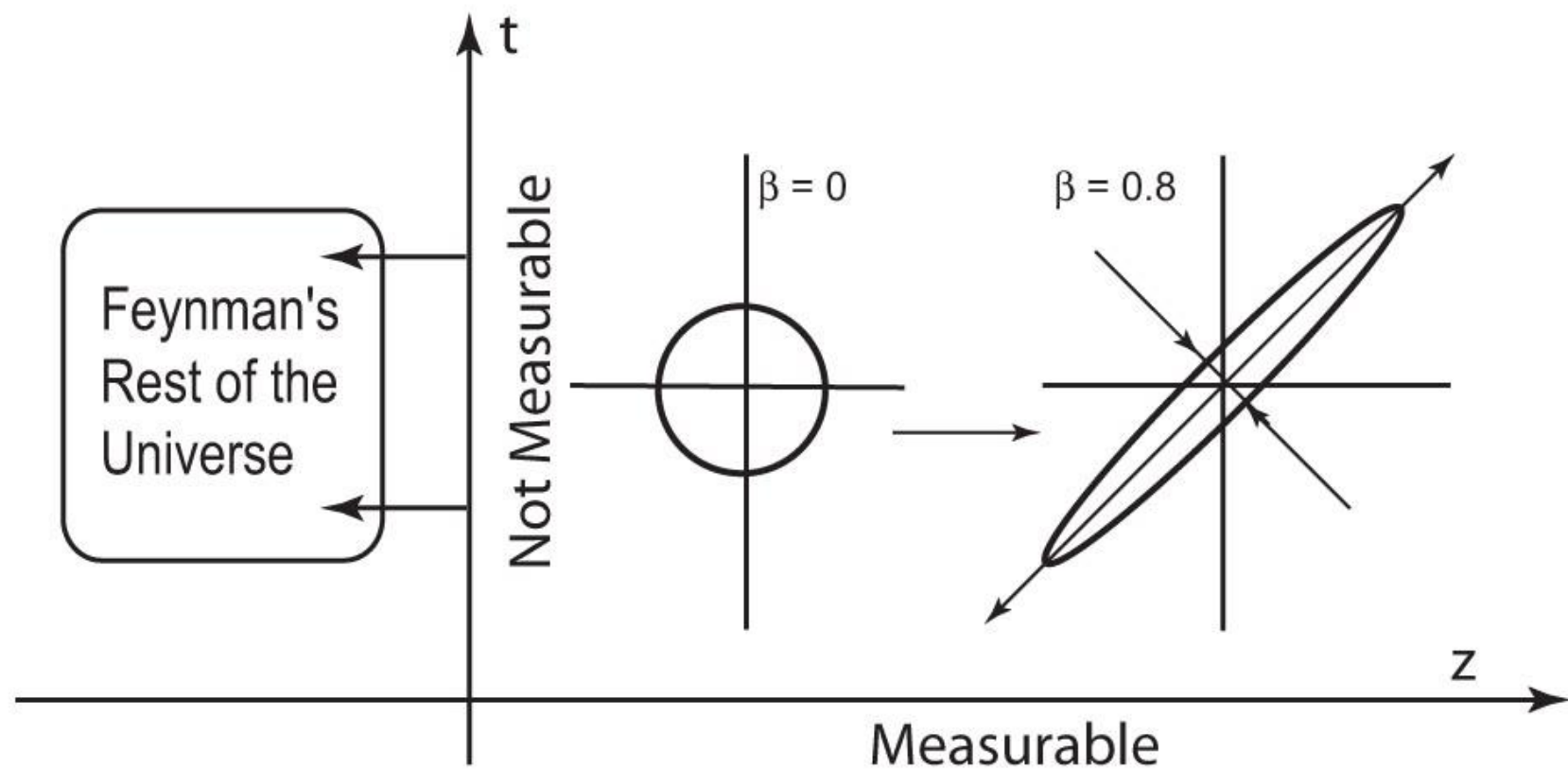
FIG. 1. Slow and fast particles. Einstein's $E = (p^2 + m^2)^{1/2}$ unifies the energy-momentum relations for massive (nonrelativistic) particles and for massless particles. The second row indicates that the little group of the Poincaré group unifies the internal space-time symmetries of massive and massless particles, as is discussed in Ref. 2. The third row states that the covariant phase-space picture of quantum mechanics forms the physical basis for the covariant harmonic-oscillator formalism which has been shown to give a unified picture of the quark model and the parton picture.

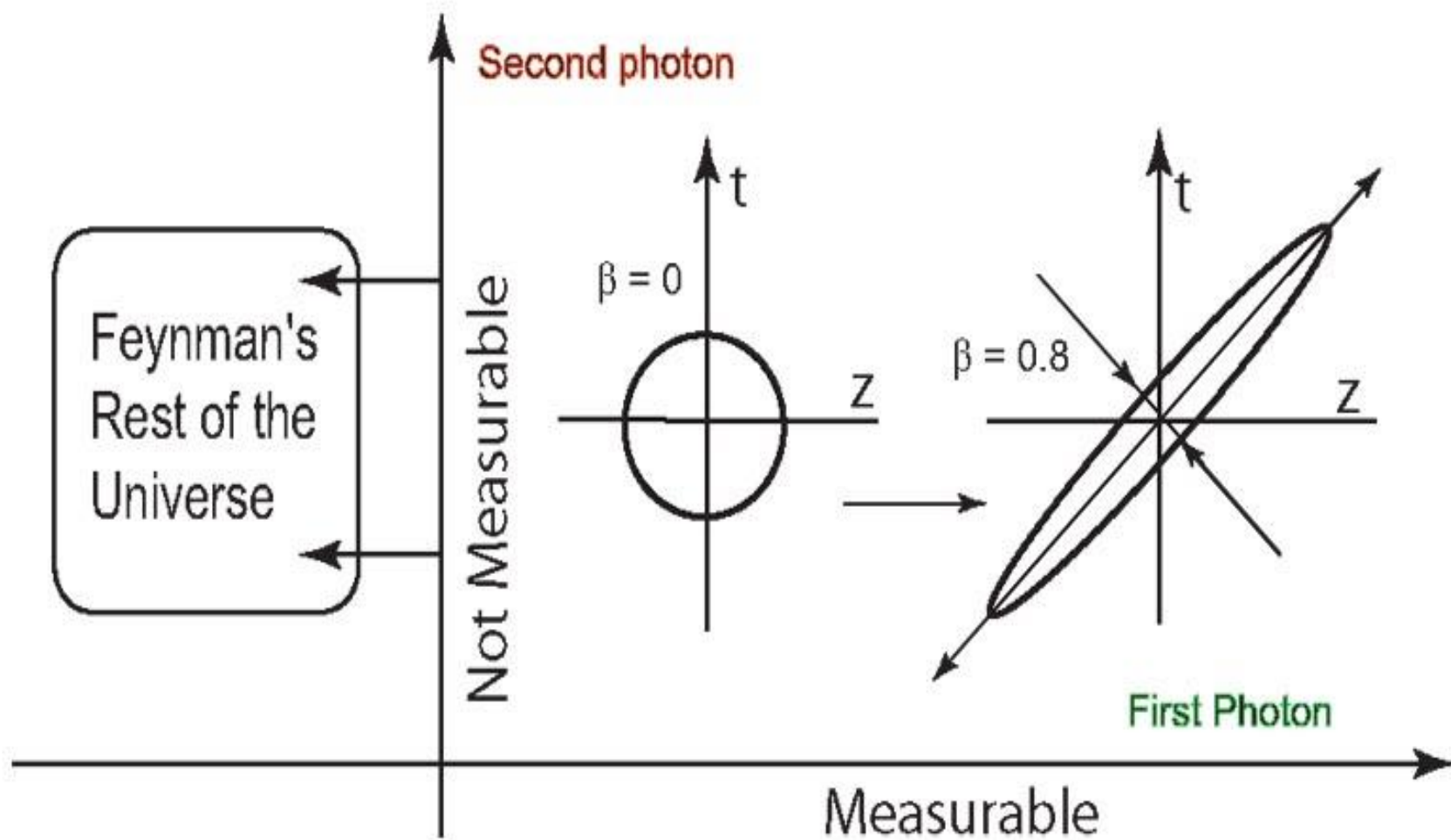
	Massive Slow	between	Massless Fast
Energy Momentum	$E = \frac{p^2}{2m}$	$E = \sqrt{m^2 + p^2}$	$E = p$
Spin, Gauge Helicity	S_3 S_1 S_2	Little Groups	S_3 Gauge Trans.
Quarks Partons	Gell-Mann's Quark Model	(Covariant Phase Space)	Feynman's Parton Model

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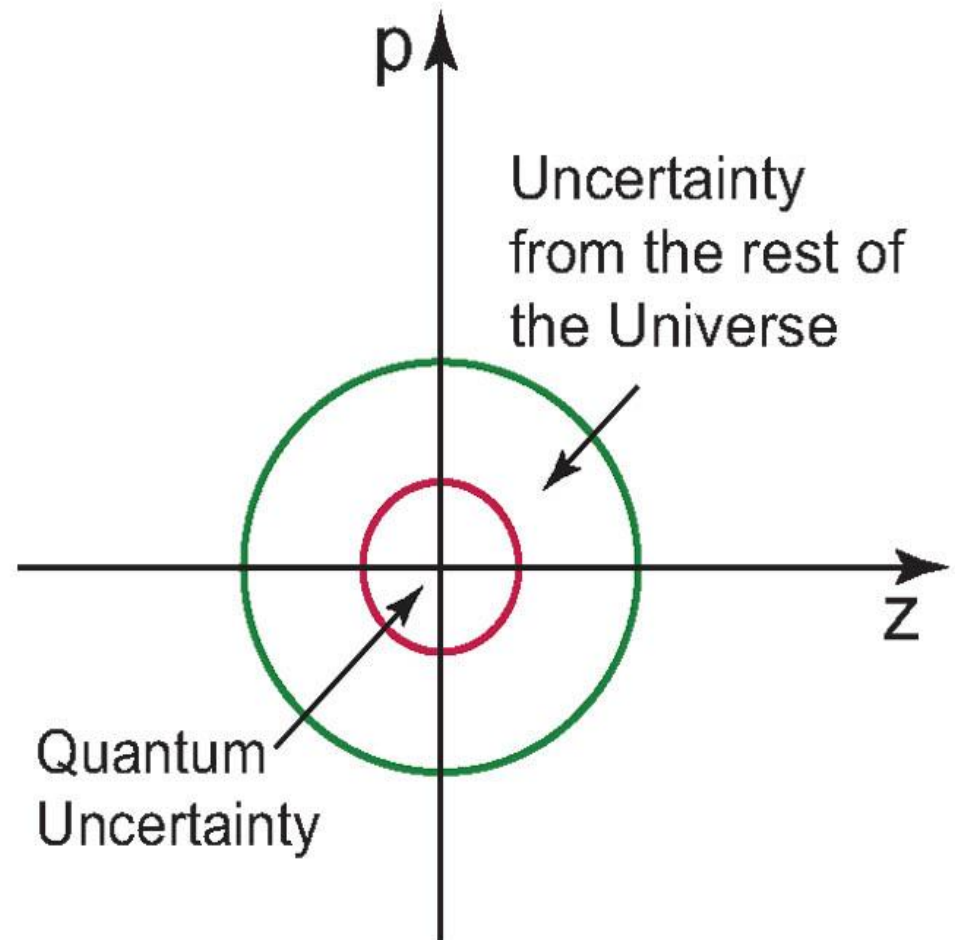






Effects of hidden variables

- Time separation as a hidden variable leads to an increase in uncertainty.
- The real issue: what is the variable hidden in the small circle.?



We started writing papers on
Feynmanism from 1973



2003



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