

Quantum Entanglement and Quantum Teleportation

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Abstract

One of the most surprising consequences of quantum mechanics is the entanglement of two or more distance particles. The “ghost” interference experiment demonstrated the astonishing nonlocal behavior of an entangled photon pair. Even though we still have question in regard to fundamental issues of the entangled quantum systems, quantum entanglement has started to play important roles in quantum information and quantum computation. Quantum teleportation is one of the hot topics. We have demonstrated a quantum teleportation experiment recently. The experimental results proved the working principle of irreversibly teleporting an unknown arbitrary quantum state from one system to another distant system by disassembling into and then later reconstructing from purely classical information and nonclassical EPR correlations. The distinct feature of this experiment is that the *complete set* of Bell states can be distinguished in the Bell state measurement. Teleportation of a quantum state can thus occur with certainty in principle.

I. INTRODUCTION

One of the most surprising consequences of quantum mechanics has been the entanglement of two or more distance particles. The two-particle entangled state was mathematically formulated by Schrödinger [1]. Consider a pure state for a system composed of two spatially separated subsystems,

$$\hat{\rho} = |\Psi\rangle \langle\Psi|, \quad |\Psi\rangle = \sum_{a,b} c(a,b) |a\rangle |b\rangle \quad (1)$$

where $\{|a\rangle\}$ and $\{|b\rangle\}$ are two sets of orthogonal vectors for subsystems 1 and 2, respectively, and $\hat{\rho}$ the density matrix. If $c(a,b)$ does not factor into a product of the form $f(a) \times g(b)$ then it follows that the state does not factor into a product state for subsystems 1 and 2:

$$\hat{\rho} \neq \hat{\rho}_1 \otimes \hat{\rho}_2 \quad (2)$$

the state was defined by Schrödinger as the entangled state.

The first classic example of a two-particle entangled state was suggested by Einstein, Podolsky, and Rosen in 1935 [2]:

$$|\Psi\rangle = \sum_{a,b} \delta(a + b - c_0) |a\rangle |b\rangle \quad (3)$$

where a and b are the momentum or the position of particle 1 and 2, respectively, c_0 is a constant. A surprising feature about the EPR state is the following: *the value of an observable (momentum or position) for neither single subsystem is determinate. However, if one of the subsystem is measured to be at a certain value for that observable the other one is 100% determined, despite the distance between the particles.* This point can be easily seen from the delta function in Eq. (3).

Another classic example of an entangled two-particle system, suggested by Bohm is a singlet state of two spin 1/2 particles [3]:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} [|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2] \quad (4)$$

where the kets $|\uparrow\rangle$ and $|\downarrow\rangle$ represent states of spin “up” and spin “down”, respectively, along an *arbitrary* direction $\hat{n}$. Again for this state, *the spin for neither particle is determined; however, if one particle is measured to be spin up along a certain direction, the other one must be spin down along that direction, despite the distance between the two spin 1/2 particles.*

The nonlocal behavior of the entangled quantum system has troubled us since 1935. In this paper we will show a “ghost” interference (section IB) experiment to demonstrate the astonishing nonlocal behavior of an entangled photon pair and provide our explanations. Even though we still have many questions in regard to the fundamental issues of the entangled quantum systems, quantum entanglement has started to play important roles in quantum information processing and quantum computing. Quantum teleportation is one of the hot topics. We will discuss a recent quantum teleportation experiment in section II.

A. Spontaneous parametric down conversion

The state of the signal-idler photon pair of spontaneous parametric down conversion (SPDC) [4] is a typical entangled EPR state. SPDC is a nonlinear optical process from which a pair of signal-idler photon is generated when a pump laser beam is incident onto an optical nonlinear crystal. Quantum mechanically, the state can be calculated by the first order perturbation theory [5] [6],

$$|\Psi\rangle = \sum_{s,i} \delta(\omega_s + \omega_i - \omega_p) \delta(\mathbf{k}_s + \mathbf{k}_i - \mathbf{k}_p) a_s^\dagger(\omega(\mathbf{k}_s)) a_i^\dagger(\omega(\mathbf{k}_i)) |0\rangle \quad (5)$$

where ω_j , $\mathbf{k}_j$ ($j = s, i, p$) are the frequency and wavevector of the signal (s), idler (i), and pump (p) respectively, ω_p and $\mathbf{k}_p$ can be considered as constants, $a_s^\dagger$ and $a_i^\dagger$ are creation operators for signal and idler photon, respectively.

The signal and idler pair is generated in such a way that the energy and momentum of neither photon is determinate; however, if one of them is measured to have certain energy

and momentum the energy and momentum of its twin is determined with certainty. It is a two-photon state, but not a state of two individual photons [7]. We have calculated the effective two-photon wavefunction for the signal-idler pair of SPDC, which may be helpful for the understanding of the entanglement nature of the state [5] [6].

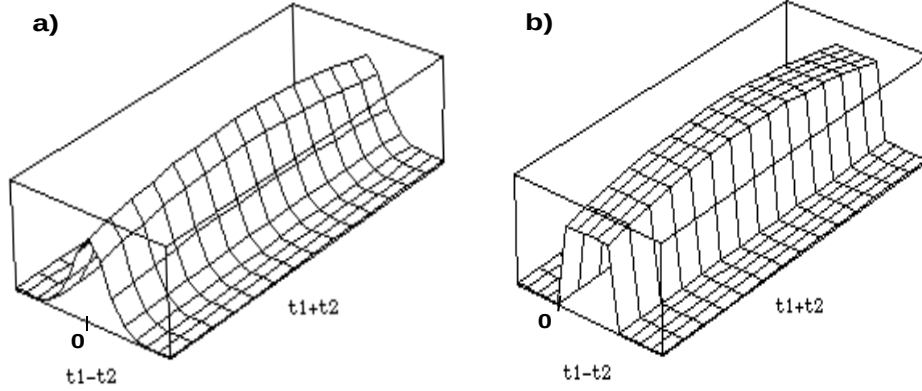


FIG. 1. Two-photon wavepacket envelopes for type-I (a) and type-II (b) SPDC. Note: type-II wavepacket has a rectangular shape in $t_1 - t_2$ and is asymmetric to $t_1 - t_2 = 0$.

The two-photon wavepacket or *biphoton* can be calculated according to the standard quantum field theory [8]. The non-factorizeable effective two-photon wavefunction $\Psi(t_1, t_2)$ ($2-D$ in configuration space) is calculated to be [5] [6] [9] [10],

$$\Psi(t_1, t_2) = A_0 e^{-\sigma_+^2 (t_1+t_2)^2} e^{-\sigma_-^2 (t_1-t_2)^2} e^{-i\Omega_s t_1} e^{-i\Omega_i t_2} \quad (6)$$

for type-I SPDC [11], where Ω_j , $j = s, i$, is the central frequency for signal or idler, $1/\sigma_{\pm}$ are “coherence times” associated with the SPDC field (σ_-) and the pump field (σ_+), $t_i \equiv T_i - r_i/c$, $i = 1, 2$, T_i is the detection time of detector i and r_i the optical pathlength from SPDC to the i th detector. For type-II SPDC [11], the wavepacket $\Psi(t_1, t_2)$, or *biphoton*, is calculated with an *asymmetrical* (respect to “zero” of $t_1 - t_2$) rectangular shape,

$$\Psi(t_1, t_2) = A_0 e^{-\sigma_+^2 (t_1+t_2)^2} \Pi(t_1 - t_2) e^{-i\Omega_s t_1} e^{-i\Omega_i t_2} \quad (7)$$

where

$$\Pi(t_1 - t_2) = \begin{cases} 1 & \text{if } 0 \leq t_1 - t_2 \leq DL \\ 0 & \text{if } \text{otherwise} \end{cases}$$

and $D \equiv 1/u_o - 1/u_e$, u_o and u_e are recognized as the group velocities of the ordinary and extraordinary rays of the SPDC crystal, and L the length of the crystal. Figure 1 is a schematic diagram of $\Psi(t_1, t_2)$ for type-I and type-II SPDC, respectively.

The signal-idler photon pair of SPDC has been widely used to prepare EPR-Bohm-Bell states for photon-polarization [12]. The four Bell states which form a complete orthonormal basis are usually represented as,

$$\begin{aligned}
|\Phi_{12}^{(\pm)}\rangle &= \frac{1}{\sqrt{2}}\{|0_1 0_2\rangle \pm |1_1 1_2\rangle\}, \\
|\Psi_{12}^{(\pm)}\rangle &= \frac{1}{\sqrt{2}}\{|0_1 1_2\rangle \pm |1_1 0_2\rangle\}.
\end{aligned} \tag{8}$$

where $|0\rangle$ and $|1\rangle$ represent the two orthogonal polarization bases, for example, $|H\rangle$ (horizontal) and $|V\rangle$ (vertical) linear polarization, respectively. In this representation the associated two-photon wavepackets (space-time) has been ignored. However, to prepare Bell states experimentally, it is very important to make the wavepackets “overlapping”, i.e., quantum mechanically indistinguishable, to keep the “superposition” in Eq. (8) valid.

B. “Ghost” interference experiment

The schematic experimental set-up of the “ghost” interference experiment is illustrated in Fig. 2 [13].

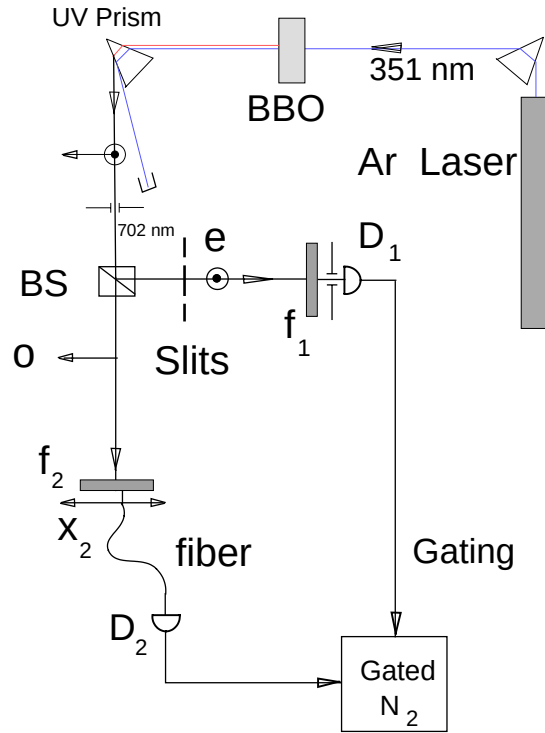


FIG. 2. Schematic set-up of the two-photon “ghost” interference-diffraction experiment.

The 351.1nm line of an Argon Ion laser is used to pump a BBO (β -BaB₂O₄) crystal which is cut at a degenerate type-II phase matching angle [11] to produce a pair of orthogonally polarized signal (e-ray of the BBO) and idler (o-ray of the BBO) photon. The pair emerges from the crystal nearly collinear, with $\omega_s \cong \omega_i \cong \omega_p/2$, where ω_j ($j = s, i, p$) are the

frequencies of the signal, idler, and pump, respectively. The pump is then separated from the down conversion by a UV grade fused silica dispersion prism and the remaining signal and idler beams are sent in different directions by a polarization beam splitting Thompson prism. The signal passes through a Young's double-slit and then travels about $1m$ to be counted by a point-like photon counting detector D_1 ($0.5mm$ in diameter). The idler travels a distance about $1.2m$ from BS to the input tip of the optical fiber. The horizontal transverse coordinate, x_2 , of the fiber input tip is scanned by an encoder driver.

Figure 3 reports a typically observed double-slit interference-diffraction pattern. The joint detection counting rate is reported as a function of x_2 , which is obtained by scanning the detector D_2 (the fiber tip) in the idler beam, whereas the double-slit is in the signal beam and D_1 is *fixed* in a symmetrical point behind the double-slit. The Young's double-slit has a slit-width $a = 0.15mm$ and slit-distance $d = 0.47mm$. The interference period is measured to be $2.7 \pm 0.2mm$ and the half-width of the envelope is estimated to be about $8mm$. By curve fitting, we conclude that the observation is a standard Young's interference pattern, i.e., a sinusoidal oscillation with a sinc function envelope:

$$R_c \propto \text{sinc}^2(x_2\pi a/\lambda z_2) \cos^2(x_2\pi d/\lambda z_2) \quad (9)$$

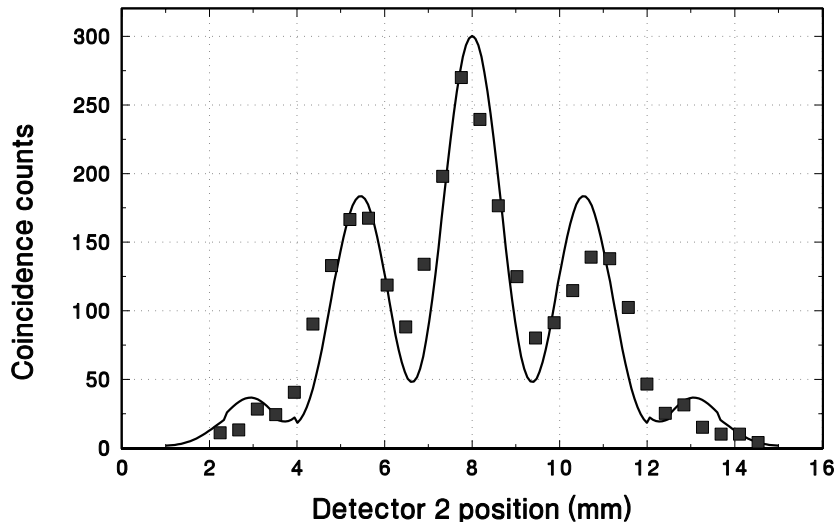


FIG. 3. Typical observed “ghost” interference-diffraction pattern: the dependence of the coincidence on the position of D_2 , which counts the idler, while the signal pass through a double-slit to be counted by the *fixed* detector D_1 . The solid curve is a calculated curve. If D_1 is moved to an asymmetrical point, which results in unequal distance to slit C and D (see fig. 4), the interference-diffraction pattern is observed to be simply shifted to one side.

The remarkable feature here is that z_2 is the distance from the slits plane, which is in the signal beam, back through BS to BBO crystal and then along the idler beam to the scanning fiber tip of detector D_2 (see Fig. 4). The calculated interference period and half-width of the *sinc* function from Eq. (9) are $2.67mm$ and $8.4mm$, respectively. Even though the

interference-diffraction pattern is observed in joint detection, the single detector counting rates are both observed to be constant when scanning detector D_1 or D_2 individually. It seems reasonable not to have any interference modulation in the single counting rate of D_2 , which is located in the “empty” idler beam. Of interest, however, is that the absence of the interference-diffraction structure in the single counting rate of D_1 , which is behind the double-slit, is mainly due to the divergence of the SPDC beam ($\gg \lambda/d$). In other words, the “blurring out” of the first order interference fringes is due to the considerably large momentum uncertainty of the signal photon. Furthermore, if D_1 is moved to an unsymmetrical point, which results in unequal distances to the two slits, the interference-diffraction pattern is observed to be simply shifted from the current symmetrical position to one side of x_2 . This is quite mind boggling: imagine that there were a first order interference pattern behind the double-slit and D_1 were moved to a completely destructive interference point (i.e. zero intensity at that point) and fixed there. Then we still can observe the same interference pattern in the joint counts (same period, shape, and counting rate), except for a phase shift!

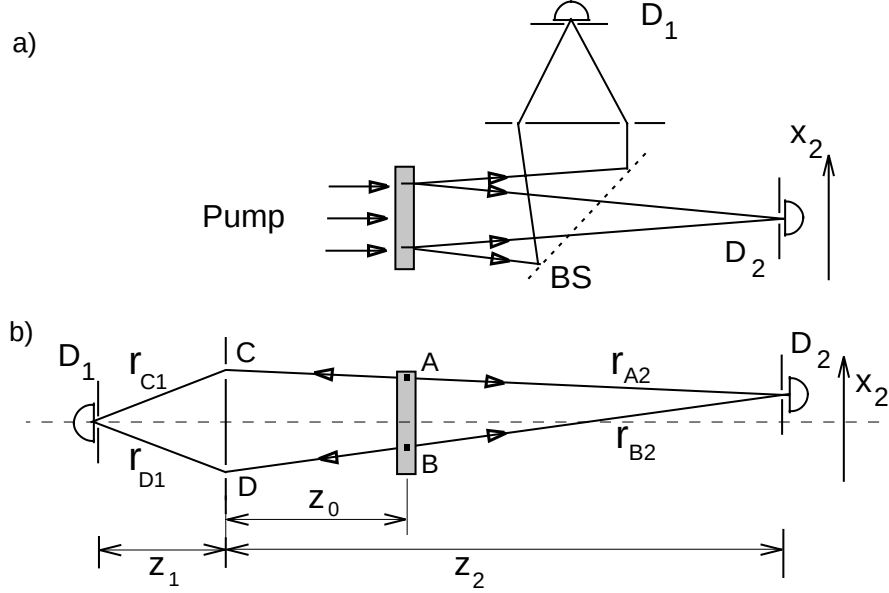


FIG. 4. Simplified experimental scheme (a) and the “unfolded” version (b).

To explain this unusual phenomenon, we present a simple quantum model. The quantum entanglement nature of the two-photon state of SPDC has been described in Eq. (5). Even though for each single photon of the pair the momentum (vector) has a considerably large uncertainty, the measurement of the momentum of either photon determines the momentum of its twin with unit probability. This important peculiarity selects the only possible optical paths in Fig. 4, when signal passes through the double-slit aperture while the other gets to D_2 . In the near degenerate case, we can simply treat the crystal as a reflecting mirror [14]. The joint counting rate R_c is determined by the probability P_{12} of detecting the signal-idler pair by detectors D_1 and D_2 simultaneously.

$$P_{12} = \langle \Psi | E_1^{(-)} E_2^{(-)} E_2^{(+)} E_1^{(+)} | \Psi \rangle = \left| \langle 0 | E_2^{(+)} E_1^{(+)} | \Psi \rangle \right|^2 \quad (10)$$

where $|\Psi\rangle$ is the two-photon state of SPDC. Let us simplify the mathematics by using the following “two-modes” expression for the state, bearing in mind that the δ functions of the “phase matching” conditions have been taken into account based on the “straight line” picture of Fig. 4.

$$|\Psi\rangle = |0\rangle + \epsilon [a_s^\dagger a_i^\dagger \exp(i\varphi_A) + b_s^\dagger b_i^\dagger \exp(i\varphi_B)] |0\rangle \quad (11)$$

where $\epsilon \ll 1$ is proportional to the pump field (classical) and the nonlinearity of the crystal, φ_A and φ_B are the phases of the pump field at A and B, and $a_j^\dagger$ ($b_j^\dagger$) are the photon creation operators for the upper (lower) mode in Fig. 4 ($j = s, i$).

In Eq. (10) the fields at the detectors are given by:

$$\begin{aligned} E_1^{(+)} &= a_s \exp(ik r_{A1}) + b_s \exp(ik r_{B1}) \\ E_2^{(+)} &= a_i \exp(ik r_{A2}) + b_i \exp(ik r_{B2}) \end{aligned} \quad (12)$$

where r_{Ai} (r_{Bi}) are the optical pathlengths from region A (B) along the upper (lower) path to the i th detector. Substituting Eqs. (11) and (12) into Eq. (10),

$$\begin{aligned} R_c \propto P_{12} &= \epsilon^2 |\exp(ik r_A + i\varphi_A) + \exp(ik r_B + i\varphi_B)|^2 \\ &\propto 1 + \cos[k(r_A - r_B)] \end{aligned} \quad (13)$$

where we assume $\varphi_A = \varphi_B$ in the second line of Eq. (13) (although this is not a necessary condition to see the interference pattern, the transverse coherence of the pump beam at A and B is crucial). In Eq. (13) we defined the overall optical lengths between the detectors D_1 and D_2 along the upper and lower paths (see Fig. 4): $r_A \equiv r_{A1} + r_{A2} = r_{C1} + r_{C2}$, $r_B \equiv r_{B1} + r_{B2} = r_{D1} + r_{D2}$, where r_{Ci} and r_{Di} are the respective pathlengths from the slits C and D to the i th detector.

If the optical paths from the fixed detector D_1 to the two slits are equal, i.e., $r_{C1} = r_{D1}$, and if $z_2 \gg d^2/\lambda$ (far field), then $r_A - r_B = r_{C2} - r_{D2} \cong x_2 d/z_2$, and Eq. (13) can be written as:

$$R_c(x_2) \propto \cos^2(x_2 \pi d / \lambda z_2) . \quad (14)$$

Equation (14) has the form of standard Young’s double-slit interference pattern. However, here again z_2 is the unusual distance from the slits plane, which is in the signal beam, back through BS to the crystal and then along the idler beam to the scanning fiber tip of detector D_2 .

If the optical paths from the fixed detector D_1 to the two slits are unequal, i.e., $r_{C1} \neq r_{D1}$, the interference pattern will be shifted from the symmetrical position from that of Eq. (14) according to Eq. (13). This interesting phenomenon has been observed and discussed following the discussion of Fig. 3.

Combine the diffraction effect by considering an integral of the effective two-photon wavefunction over the slit width (the superposition of infinite number of probability amplitudes results in a click-click joint detection event), the interference-diffraction coincidence counting rate for the double-slit case is given by:

$$R_c(x_2) \propto \text{sinc}^2(x_2\pi a/\lambda z_2) \cos^2(x_2\pi d/\lambda z_2) \quad (15)$$

which is exactly the same as Eq. (9) obtained from experimental data fittings. If the finite size of the detectors and the divergence of the pump are taken into account, the interference visibility will be reduced. These factors have been considered in the theoretical plots of Fig. 3.

II. QUANTUM TELEPORTATION EXPERIMENT

Quantum information processing is based on the use of quantum - qubit. This makes it very different from classical information processing. First, since a quantum will be destroyed after a measurement, there is only one chance for measurement. However, one measurement of a quantum would not, in principle, provide any information about the state of the qubit according to quantum mechanics. How to learn the state of a quantum? Can we make many copies and then make an ensemble measurement of the copies? Unfortunately, no-cloning theorem said: NO! Quantum mechanics does not allow a “copier” to duplicate copies of a quantum.

How to transport the state of a quantum? Quantum teleportation might be the unique approach.

The idea of quantum teleportation is to utilize the nonlocal correlations between an Einstein-Podolsky-Rosen pair of particles [2] to prepare a quantum system in some state, which is the exact replica of an arbitrary unknown state of a distant individual quantum system [15]. Three experiments in this direction were published recently [16] [17] [18].

The following conditions must be satisfied in any claim for quantum teleportation: (i) the input quantum state, which is teleported in the experiment must be an *arbitrary* state, (ii) there must be an output quantum state which is an “instantaneous copy” of the input quantum state, (iii) for any input state, the teleportation must be deterministic, not “statistical”, and (iv) the Bell state measurement (BSM) must be able to distinguish the complete set of the orthogonal Bell states so that the input state can be teleported with certainty.

We have recently demonstrated a quantum teleportation scheme which satisfies all four of the above conditions [19]. The input state is an *arbitrary polarization state* and the BSM can distinguish the *complete set* four orthogonal Bell states. Teleportation of a quantum state can thus occur with certainty in principle. This is because the BSM is based on nonlinear interactions which are *necessary* and *non-trivial* physical processes for quantum teleportation [20] [21]. Another point we like to emphasize is that the Bell state measurement is deterministic in this scheme. This is important for the teleportation of a qubit, which is “carried” by a single quantum.

The basic elements of the experiment are schematically shown in Fig. 5. Just as the original proposal of quantum teleportation [15], it consists of four essential parts: (a) the input state, (b) the EPR pair, (c) Alice (who performs the BSM of the input state and her EPR particle), and (d) Bob (who carries out unitary operations on his EPR particle). The input quantum state is an *arbitrary polarization state* given by,

$$|\Psi_1\rangle = \alpha|0_1\rangle + \beta|1_1\rangle, \quad (16)$$

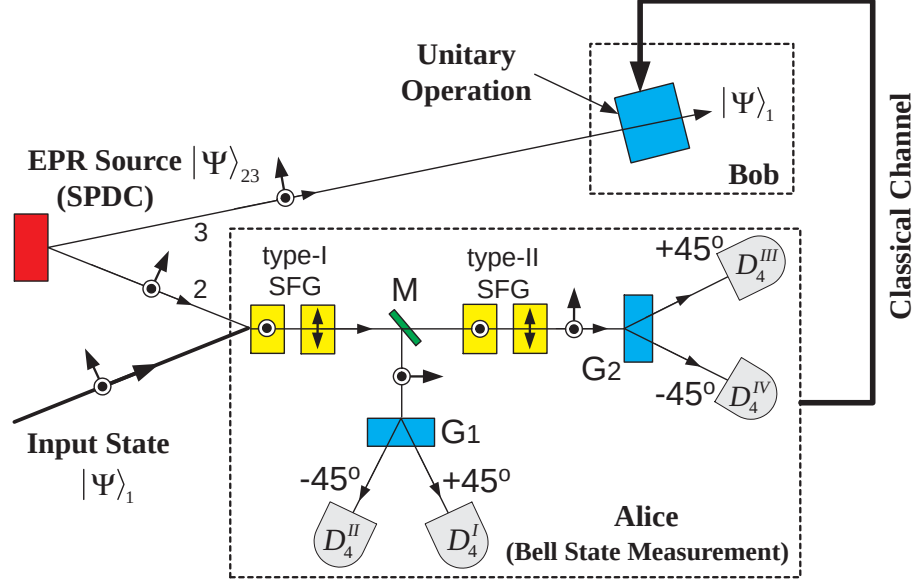


FIG. 5. Schematic of quantum teleportation with complete BSM. Nonlinear interactions (SFG) are used to perform the BSM. $\odot$ and $\uparrow$ represent the respective horizontal and vertical orientations of the optic axes of the crystals.

where $|0\rangle$ and $|1\rangle$ represent the two orthogonal linear polarization bases (specifically in this paper) $|H\rangle$ (horizontal) and $|V\rangle$ (vertical) respectively. α and β are two arbitrary complex amplitudes with respect to the $|0\rangle$ and $|1\rangle$ bases and they satisfy the condition $|\alpha|^2 + |\beta|^2 = 1$. Note, α and β are allowed to be functions of time t , $\alpha(t)$ and $\beta(t)$, in our experiment. The EPR pair shared by Alice and Bob is prepared by spontaneous parametric down conversion (SPDC) as,

$$|\Psi_{23}\rangle = \frac{1}{\sqrt{2}}\{|0_2 0_3\rangle - |1_2 1_3\rangle\}, \quad (17)$$

with the subscripts 2 and 3 as labeled in Fig. 5 [22]. The complete state of the three particles before Alice's measurement is then,

$$|\Psi_{123}\rangle = \frac{\alpha}{\sqrt{2}}\{|0_1 0_2 0_3\rangle - |0_1 1_2 1_3\rangle\} + \frac{\beta}{\sqrt{2}}\{|1_1 0_2 0_3\rangle - |1_1 1_2 1_3\rangle\}. \quad (18)$$

State (18) can now be re-written in the following form based on the complete set of the orthonormal Bell states of Eq. (8),

$$\begin{aligned} |\Psi_{123}\rangle = & \frac{1}{2}\{|\Phi_{12}^{(+)}\rangle(\alpha|0_3\rangle - \beta|1_3\rangle) + |\Phi_{12}^{(-)}\rangle(\alpha|0_3\rangle + \beta|1_3\rangle) \\ & + |\Psi_{12}^{(+)}\rangle(-\alpha|1_3\rangle + \beta|0_3\rangle) + |\Psi_{12}^{(-)}\rangle(-\alpha|1_3\rangle - \beta|0_3\rangle)\}. \end{aligned} \quad (19)$$

To teleport the state of particle 1 to particle 3 reliably, Alice must be able to distinguish her four Bell states by means of the BSM performed on particle 1 and her EPR share (particle

2). She then tells Bob through a classical channel to perform a corresponding linear unitary operation on his EPR share (particle 3) to obtain an exact replica of the state of particle 1. This completes the process of quantum teleportation.

The distinct feature of the scheme shown in Fig. 5 is that the BSM is based on nonlinear interactions: optical Sum Frequency Generation (SFG) (or “upconversion”). Four SFG nonlinear crystals are used for “measuring” and “distinguishing” the complete set of the four Bell states. Photon 1 and photon 2 may interact either in the two type-I crystals or in the two type-II crystals to generate a higher frequency photon (labeled as photon 4). The projection measurements on photon 4 (either the 45° or 135° direction) correspond to the four Bell states of photon 1 and photon 2, $|\Phi_{12}^{(\pm)}\rangle$ and $|\Psi_{12}^{(\pm)}\rangle$.

Let us now discuss the BSM in detail (see Fig. 5). The first type-I SFG crystal converts two $|V\rangle$ polarized photons $|1_1 1_2\rangle$ into a single horizontal polarized photon $|H_4\rangle$. Likewise, the second type-I SFG crystal converts two $|H\rangle$ polarized photons $|0_1 0_2\rangle$ into a single vertical polarized photon $|V_4\rangle$. The first and the last terms on the right-hand side in Eq. (18) thus become,

$$|\Psi_{43}\rangle = \alpha|V_4 0_3\rangle - \beta|H_4 1_3\rangle.$$

Dichroic beamsplitter M reflects only SFG photons to the 45° polarization projector G_1 . Two detectors D_4^I and D_4^{II} are placed at the 45° and 135° output ports of G_1 respectively. Denoting the 45° and 135° polarization bases by $|45^\circ\rangle$ and $|135^\circ\rangle$, the state $|\Psi_{43}\rangle$ can be re-written as,

$$|\Psi_{43}\rangle = \frac{1}{\sqrt{2}}\{|45^\circ\rangle_4(\alpha|0_3\rangle - \beta|1_3\rangle) + |135^\circ\rangle_4(\alpha|0_3\rangle + \beta|1_3\rangle)\}, \quad (20)$$

which gives,

$$\begin{aligned} |\Psi\rangle_{3|D_4^I} &= \alpha|0_3\rangle - \beta|1_3\rangle, \\ |\Psi\rangle_{3|D_4^{II}} &= \alpha|0_3\rangle + \beta|1_3\rangle, \end{aligned} \quad (21)$$

i.e., if detector D_4^I (45°) is triggered, the quantum state of Bob’s EPR share (photon 3) is: $|\Psi_3\rangle = \alpha|0_3\rangle - \beta|1_3\rangle$ and if detector D_4^{II} (135°) is triggered, the quantum state of Bob’s photon is: $|\Psi_3\rangle = \alpha|0_3\rangle + \beta|1_3\rangle$.

As we have analyzed above, the 45° and the 135° polarized type-I SFG components in Eq. (20) correspond to the superposition of $|0_1 0_2\rangle$ and $|1_1 1_2\rangle$ which are the respective Bell states $|\Phi_{12}^{(+)}\rangle$ and $|\Phi_{12}^{(-)}\rangle$.

Similarly, the other two Bell states are distinguished by the type-II SFG’s. The states $|0_1 1_2\rangle$ and $|1_1 0_2\rangle$ are made to interact in the first and the second type-II SFG crystals respectively to generate a higher frequency photon with either horizontal (the first type-II SFG) or vertical (the second type-II SFG) polarization. A 45° polarization projector G_2 is used after the type-II SFG crystals and two detectors D_4^{III} and D_4^{IV} are placed at the 45° and the 135° output ports of G_2 respectively. On the new bases of 45° and 135° for the SFG photon, the second and the third terms on the right-hand side in Eq. (18) thus become,

$$|\Psi_{43}\rangle = \frac{1}{\sqrt{2}}\{|45^\circ\rangle_4(-\alpha|1_3\rangle + \beta|0_3\rangle) + |135^\circ\rangle_4(-\alpha|1_3\rangle - \beta|0_3\rangle)\}, \quad (22)$$

which gives,

$$\begin{aligned} |\Psi\rangle_{3|D_4^{III}} &= -\alpha|1_3\rangle + \beta|0_3\rangle, \\ |\Psi\rangle_{3|D_4^{IV}} &= -\alpha|1_3\rangle - \beta|0_3\rangle, \end{aligned} \quad (23)$$

i.e., if detector D_4^{III} (45°) is triggered, the quantum state of Bob's photon is: $|\Psi_3\rangle = -\alpha|1_3\rangle + \beta|0_3\rangle$ and if detector D_4^{IV} (135°) is triggered, the quantum state of Bob's photon is: $|\Psi_3\rangle = -\alpha|1_3\rangle - \beta|0_3\rangle$.

The 45° and the 135° polarized type-II SFG components correspond to the superposition of $|0_1 1_2\rangle$ and $|1_1 0_2\rangle$ which are the Bell states $|\Psi_{12}^{(+)}\rangle$ and $|\Psi_{12}^{(-)}\rangle$ respectively.

To obtain the exact replica of the state of Eq. (16), Bob needs simply to perform a corresponding unitary transformation after learning from Alice which of her four detectors, D_4^I , D_4^{II} , D_4^{III} , or D_4^{IV} , is triggered at time t [23].

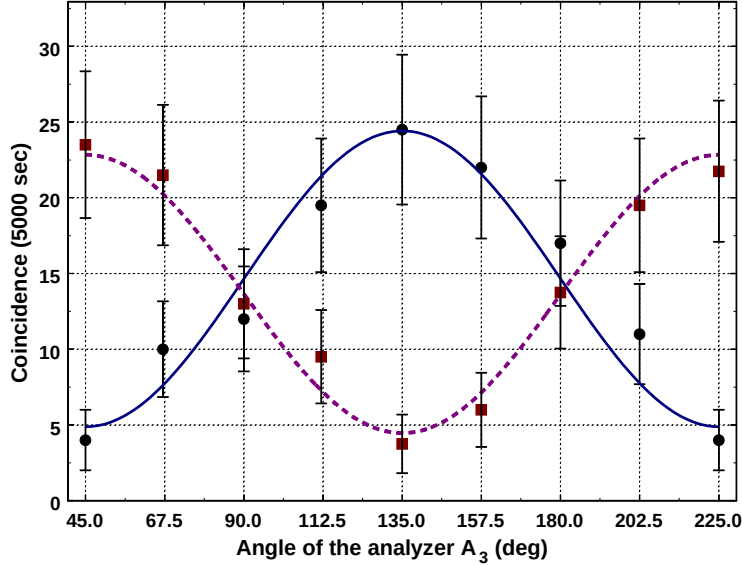


FIG. 6. Solid line (circled data points) is the joint detection rate D_4^I - D_3 . Dashed line (square data points) is for D_4^{II} - D_3 . The expected π phase shift is clearly demonstrated.

To demonstrate the working principle of this scheme, we measure the joint detection rates between detectors D_4^I - D_3 , D_4^{II} - D_3 , D_4^{III} - D_3 and D_4^{IV} - D_3 , where D_3 is Bob's detector (see Fig. 8). In these measurements we choose the input state $|\Psi_1\rangle$ as a linear polarization state. For a fixed input polarization state, the angle of the polarization analyzer A_3 which is placed in front of Bob's detector is rotated and the joint detection rates are recorded. Figure 6 show two typical data sets for D_4^I - D_3 and D_4^{II} - D_3 . The input polarization state is 45° . Clearly, these data curves confirm Eq. (21). The different phases of the two curves reflect the phase difference between the two states in Eq. (21). As expected, experimental data for D_4^{III} - D_3 and D_4^{IV} - D_3 show similar behavior, see Fig. 7.

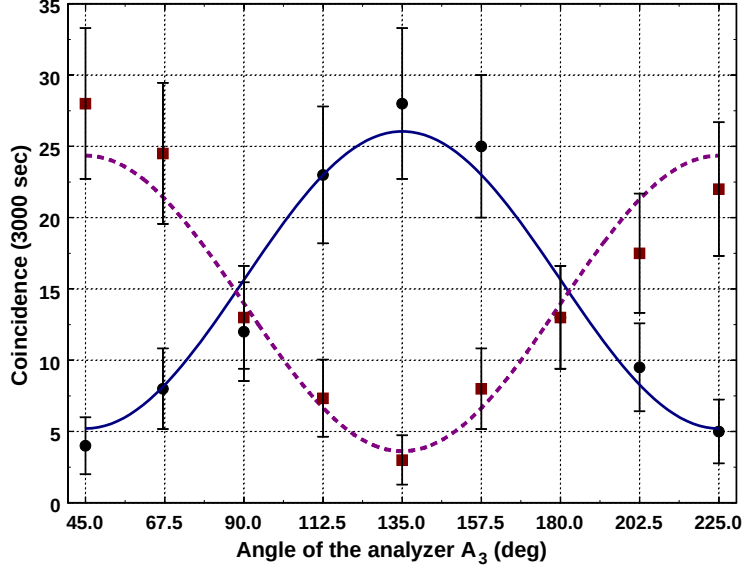


FIG. 7. Solid line (circled data points) is the joint detection rate $D_4^{III} - D_3$. Dashed line (square data points) is for $D_4^{IV} - D_3$. Again, the expected π phase shift is clearly demonstrated.

The schematic of the experimental setup is shown in Fig. 8. The input polarization state is prepared by using a $\lambda/2$ plate from a femtosecond laser pulse (pulse width ≈ 100 fs and central wavelength = 800nm) [24]. The EPR pair (730nm-885nm photon pair) is generated by two non-degenerate type-I SPDC's. The optical axes of the first and the second SPDC crystals are oriented in the respective horizontal ($\odot$) and vertical ($\updownarrow$) directions. The SPDC crystals are pumped by a 45° polarized 100fs laser pulse with 400nm central wavelength. Since the two crystals are pumped equally, the SPDC pair can be generated either in the first BBO as $|V_{885}\rangle_2|V_{730}\rangle_3$ ($|1_21_3\rangle$) or in the second BBO as $|H_{885}\rangle_2|H_{730}\rangle_3$ ($|0_20_3\rangle$) with equal probability (885 and 730 refer to the wavelengths in nanometer).

In order to prepare an EPR state in the form of Eq. (17) (a Bell state), these two amplitudes have to be quantum mechanically “indistinguishable” and have the expected relative phase. A Compensator (C-1) is used for this purpose and it consists of two parts: a thick quartz rod and two thin plates. The thick quartz rod is used to compensate the time delay between the two amplitudes $|1_21_3\rangle$ and the $|0_20_3\rangle$, and two thin quartz plates are used to adjust the relative phase between them by angular tilting. A dichroic beamsplitter DBS is placed behind the SPDC crystals to separate and send the photon 2 (885nm) and photon 3 (730nm) to Alice and Bob respectively. To check the EPR state, a flipper mirror FM is used to send the photon 2 (885nm) to a photon-counting detector D_2 for EPR correlation measurement. Both the space-time and polarization correlations must be checked before teleportation measurements, in order to be certain of having high degree EPR entanglement and the expected relative phase between the $|1_21_3\rangle$ and the $|0_20_3\rangle$ amplitudes (see Ref. [25] for details). Once the EPR state in Eq. (17) is prepared, FM is flipped-down and photon 2 (885nm) is given to Alice for BSM with photon 1.

The BSM consists of four SFG nonlinear crystals, two 45° projectors (G_1 and G_2), four single photon counting detectors (D_4^I , D_4^{II} , D_4^{III} , and D_4^{IV}) and two compensators as well as

other necessary optical components. The working principle of the BSM has been discussed previously.

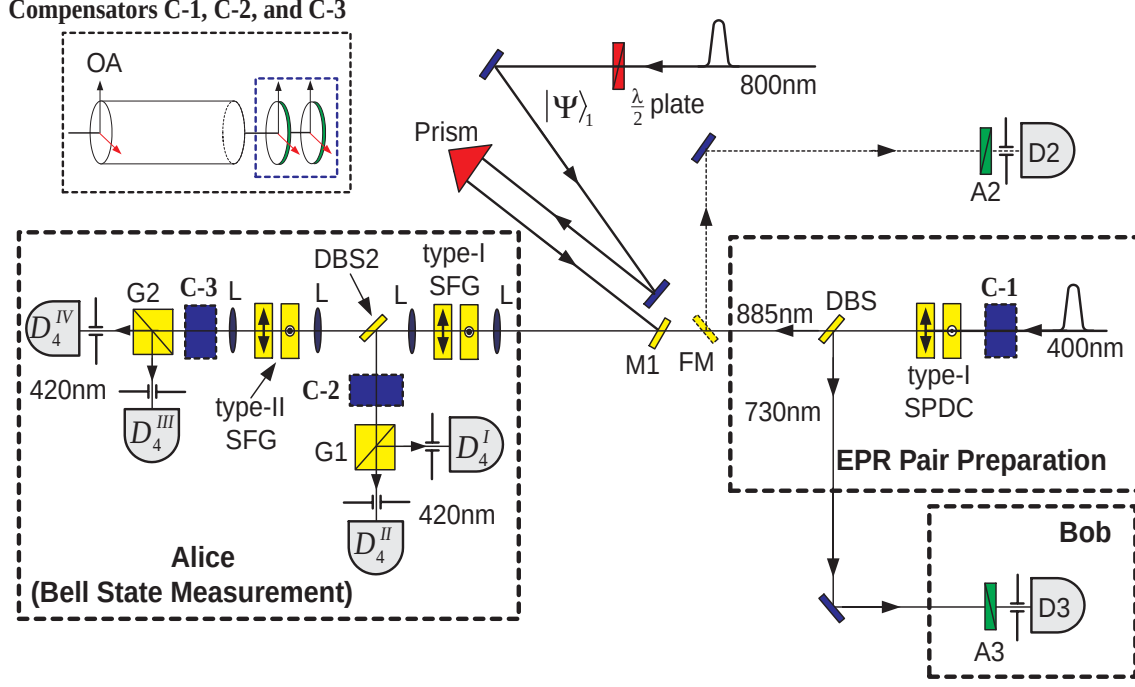


FIG. 8. Schematic of the Experimental Setup. The inset shows the details of the compensators. See text for details.

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- [1] E.Schrödinger, *Naturwissenschaften* **23**, (1935) 807, 823, 844; translations appear in *Quantum Theory and Measurement*, ed. J.A. Wheeler and W.H. Zurek, Princeton University Press, New York, (1983).
 - [2] A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, (1935) 777.
 - [3] D. Bohm, *Quantum Theory*, Prentice Hall Inc., New York, (1951).
 - [4] D.N. Klyshko, *Photon and Nonlinear Optics*, Gordon and Breach Science, New York, (1988); A. Yariv, *Quantum Electronics*, John Wiley and Sons, New York, (1989). "Spontaneous Parametric Down Conversion" was called "Spontaneous Fluorescence" and "Spontaneous Scattering" by the pioneer workers.
 - [5] M.H. Rubin, D.N. Klyshko, and Y.H. Shih, *Phys. Rev. A* **50**, (1994) 5122.
 - [6] Y.H. Shih, *Advances in Atomic, Molecular, and Optical Physics*, ed., B. Bederson and H. Walther, Academic Press, Cambridge, 1997.
 - [7] T.B. Pittman *et al.*, *Phys. Rev. Lett.*, **77**, (1996) 1917; D.V. Strekalov, T.B. Pittman, and Y.H. Shih, *Phys. Rev. A*, **57**, (1998) 567.

- [8] R.J. Glauber, Phys. Rev. **130**, (1963) 2529; **131**, (1963) 2766.
- [9] Y.H. Shih and A.V. Sergienko, Phys. Rev. A, **50**, (1994) 2564.
- [10] A.V. Sergienko, Y.H. Shih, and M.H. Rubin, JOSAB, **12**, (1995) 859.
- [11] In type-I SPDC, signal and idler are both ordinary (or extraordinary) rays of the SPDC crystal; however, in type-II SPDC the signal and idler are orthogonally polarized, i.e., one is ordinary and the other is extraordinary.
- [12] C.O. Alley and Y.H. Shih, *International Symposium on Foundations of Quantum Mechanics in the Light of New Technology*, edited by M. Namiki *et al*, (1986) p. 47; Y.H. Shih and C.O. Alley, Phys. Rev. Lett., **61**, (1988) 2921; Z.Y. Ou and L. Mandel, Phys. Rev. Lett., **62**, (1988) 50; T.E. Kiess *et al.*, Phys. Rev. Lett., **71**, (1993) 3893; P.G. Kwiat *et al.*, Phys. Rev. Lett., **75**, (1995) 4337; Kwiat *et al.*, Phys. Rev. A, **60**, (1999) R773; Y.-H. Kim, S.P. Kulik, and Y.H. Shih, quant-ph/0007067, submitted to Phys. Rev. Lett..
- [13] D.V. Strekalov, A.V. Sergienko, D.N. Klyshko, and Y.H. Shih, Phys. Rev. Lett., **74**, (1995) 3600.
- [14] For related theory see, D.N. Klyshko, Phys. Lett. A **132**, (1988) 299; Sov. Phys. Usp., **31**, (1988) 74; A.V. Belinskii and Klyshko, JETP, **78**, (1994) 259.
- [15] C.H. Bennett *et al.*, Phys. Rev. Lett. **70**, (1993) 1895.
- [16] D. Bouwmeester *et al.*, Nature **390**, (1997) 575.
- [17] D. Boschi *et al.*, Phys. Rev. Lett. **80**, (1998) 1121.
- [18] A. Furusawa *et al.*, Science **282**, (1998) 706.
- [19] Y.-H. Kim, S.P. Kulik, and Y.H. Shih, Submitted to Phys. Rev. Lett., (2000).
- [20] L. Vaidman and N. Yoran, Phys. Rev. A **59**, (1999) 116; N. Lütkenhaus, J. Calsamiglia, and K.-A. Suominen, *ibid.* **59**, (1999) 3295.
- [21] D.N. Klyshko, JETP **87**, (1998) 639.
- [22] Note that any one of the four Bell states can be used for this purpose.
- [23] Complete BSM using SFG is also useful for other applications, see C.H. Bennett and S.J. Wiesner, Phys. Rev. Lett. **69**, (1992) 2881.
- [24] In the current experiment, we use a femtosecond laser pulse as the input. Only one out of approximately 10^{10} photons, all in the same polarization state, in each laser pulse is actually “upconverted” in the SFG process. What is being teleported is the state or qubit associated with this photon. In principle, it does not prevent one to use a single-photon qubit as the input state in this experiment. Due to the low efficiency of SFG, one needs to wait a much longer time for teleportation to occur. The efficiency of SFG is roughly estimated to be 0.1~1%.
- [25] Y.-H. Kim, S.P. Kulik, and Y.H. Shih, Phys. Rev. A **62**, (2000) 011802(R); quant-ph/0007067.